

Gravitational Collapse, Negative World and Complex Holism*

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Abstract

Building on the engine-pump paradigm of ChaNoXity, this paper argues that complex holism — as the competitive homeostasis of dispersion and concentration — is the operating mode of Nature. Specifically, we show that the negative world \mathfrak{W} is a gravitationally collapsed black hole that was formed at big-bang time $t = 0$ as the pair (W, \mathfrak{W}) , with W a real world, and gravity the unique expression of the maximal multifunctional nonlinearity of the negative world \mathfrak{W} in the functional reality of W . The temperature of a gravitationally collapsed system does enjoy the relationship $T \propto 1/r$ with its radius, but the entropy follows the usual volumetric alignment with microstates, reducing to the surface approximation only at small r . It is not clear if quantum non-locality is merely a linear manifestation of complex holism, with the interaction of quantum gates in quantum entanglements resulting in distinctive features from the self-evolved structures of complex holism remaining an open question for further investigation.

Keywords: Chaos-Nonlinearity-compleXity, “Capital”-“Culture”-Holism, competitive collaboration, Critical and Triple points, phase transition, negative world, economic holism.

1 Introduction

In a recent two-part discourse [29], a rigorous, scientific, self-contained, and unified formulation of complex holism has been developed. Science of the last 400 years has essentially evolved by the reductionist tools of linear mathematics in which a composite whole is regarded as the sum of its component parts. Increasingly however, a realization has grown that most of the important manifestations of nature in such diverse fields as ecology, biology, social, economic and the management sciences, beside physics and cosmology, display a holistic behaviour which, simply put, is the philosophy that parts of any whole cannot exist and be understood except in their relation to the whole: the system as a whole determines in an important way how the parts behave. These complex self-organizing systems evolve on emergent feedback mechanisms and processes that “interact with themselves and produce themselves from themselves”: they are “more than the sum of their parts”. Thus society is more than a collection of individuals, life is more than a mere conglomeration of organs as much as human interactions are rarely dispassionate.

Living organisms require both energy and matter to continue living, are composed of at least one cell, are homeostatic, and evolve; life organizes matter into increasingly complex forms in apparent violation of the Second Law of Thermodynamics that forbids order in favour of discord, instability and lawlessness; in fact “a living organism continually increases its entropy and thus tends to approach the dangerous state of maximum entropy, which is *death*”. However, “It can only keep aloof from it, i.e. stay *alive*, by continually drawing from its environment “*negative entropy*”. It thus maintains itself stationery at a fairly high level of orderliness (= fairly low level of entropy) (by) continually sucking orderliness from its environment” [25]. Holism entails “life (to be) a far-from-equilibrium dissipative structure that maintains its local level of self organization at the cost of increasing the entropy of the larger global system in which the structure is imbedded” [24], “a living individual is defined within the cybernetic paradigm as a system of inferior negative feedbacks subordinated to (being at the service of) a superior positive feedback” [16], “life is a balance between the imperatives of survival and energy degradation” [7], and “life is a special complex system of activating mind

*To appear in *Nonlinear Analysis B: Real World Applications*. Received : April 22, 2010, Accepted: May 04, 2010

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and restraining body” [28] identifiable respectively by an anti-thermodynamic backward and thermodynamic forward arrows.

The linear reductionist nature of present mainstream science raises many deep-rooted and fundamental questions that apparently defy logical interpretation within its own framework; as do questions involving socio-economic, collective (as opposed to individualistic), and biological relations. The issues raised by this dichotomy have been well known and appreciated for long enough, leading often to bitter and acrimonious debate between protagonists of the reductionist and holistic camps: ChaNoXity [29] aims at integrating Chaos-Nonlinearity-compleXity into the unified structure of holism that has been able to shed fresh insight to these complex manifestations of Nature. The characteristic features of holism are self-organization and emergence: Self-organization involves the internal organization of an open system to increase from numerous nonlinear interactions among the lower-level hierarchical components *without being guided or managed from outside*. The rules specifying interactions among the system’s components are executed using only local information, without reference to the global pattern. *Self-organization* relies on three basic ingredients: positive-negative feedbacks, exploitation-exploration, and multiple interactions. In *emergence*, global-level coherent structures, patterns and properties arise from nonlinearly interacting local-level processes. The structures and patterns cannot be understood or predicted from the behavior or properties of the components alone: the global patterns cannot be reduced to individual behaviour. Emergence involves multi-level systems that interact at both higher and lower level; these emergent systems in turn exert both upward and downward causal influences.

Complexity results from the interaction between parts of a system such that it manifests properties not carried by, or dictated by, individual components. Thus complexity resides in the interactive competitive collaboration¹ between the parts; the properties of a system with complexity are said to “emerge, without any guiding hand”. A complex system is an assembly of many interdependent parts, interacting with each other through competitive nonlinear collaboration, leading to self-organized, emergent holistic behaviour.

What is chaos? Chaos theory describes the behavior of dynamical systems — systems whose states evolve with time — that are highly sensitive to initial conditions. This sensitivity, expressing itself as an exponential growth of perturbations in initial conditions, render the evolution of a chaotic system appear to be random, although these are fully deterministic systems with no random elements involved. Chaos responsible for complexity [27] is the eventual outcome of *non-reversible* iterations of one-dimensional *non-injective* maps; noninjectivity leads to irreversible nonlinearity and one-dimensionality constrains the dynamics to evolve with the minimum spatial latitude thereby inducing emergence of new features as required by complexity. In this sense chaos is the maximal ill-posed irreversibility of the maximal degeneracy of multifunctions; features that cannot appear through differential equations. The mathematics involve topological methods of convergence of nets and filters² with the multifunctional graphically converged adherent sets effectively enlarging the functional space in the outward manifestation of Nature. Chaos therefore is more than just an issue of whether or not it is possible to make accurate long-term predictions of the system: chaotic systems are necessarily sensitive to initial conditions and topologically mixing with dense periodic orbits; this, however, is not sufficient, and maximal ill-posedness of solutions is a prerequisite for the evolution of complex structures.

ChaNoXity involves a new perspective of the dynamical evolution of Nature based on the irreversible multifunctional multiplicities generated by the equivalence classes from iteration of noninvertible maps, eventually leading to chaos of maximal ill-posedness, [27]. The iterative evolution of difference equations is in sharp contrast to the smoothness, continuity, and reversible development of differential equations which cannot lead to the degenerate irreversibility inherent in the equivalence classes of ill-posed systems. Unlike evolution of differential equations, difference equations update their progress at each instant with reference to its immediate predecessor, thereby satisfying the crucial requirement of adaptability and experience based learning that constitutes the distinctive feature of complex systems. Rather than the smooth continuity of differential equations, Nature takes advantage of jumps, discontinuities, and singularities to choose from the vast multiplicity of possibilities that rejection of such regularizing constraints entail. Non-locality and holism, the natural consequences of this paradigm, are to be compared with the reductionist determinism of classical Newtonian reversibility suggesting striking formal correspondence with superpositions, qubits and entanglement of quantum theory [29]. Complex holism is to be understood as complementing mainstream simple reductionism — linear science has after all stood the test of the last 400 years as quantum mechanics is acknowledgedly one of the most successful yet possibly among the most mysterious of scientific theories. Its success lies in the capacity to classify and predict the physical world — the mystery in what this physical

¹Competitive collaboration — as opposed to reductionism — in the context of this characterization is to be understood as follows: *The interdependent parts retain their individual identities, with each contributing to the whole in its own characteristic fashion within a framework of dynamically emerging global properties of the whole.* Although the properties of the whole are generated by the parts, the individual units acting independently on their own cannot account for the global behaviour of the total.

²These are generalizations of the usual concept of sequences and, in what follows, may be read as such.

world must be like for it to be as it is supposed to be.

2 ChaNoXity: The New Science of Complex Holism

The mathematical structure of ChaNoXity is based on the discrete evolution of difference equations rather than on the smooth and continuous unfolding of differential equations. The fundamental goal of chanoxity is to suggest, justify and institute the existence of an anti-thermodynamic arrow that allows open systems the privilege of metaphorically “sucking orderliness from the environment” and thereby survive in the highly improbable state of being “alive”. For an exhaustive account of the very brief overview recounted below, reference should be made to [27, 29].

2.1 Mathematics of ChaNoXity: Nonlinearity, Multiplicity, Non-smoothness [11, 12, 18, 19]

(A) Topologies. (i) If \sim is an equivalence relation on a set X , the class of all saturated sets $[x]_{\sim} = \{y \in X : y \sim x\}$ is a topology on X ; this *topology of saturated sets* constitutes the defining topology of chaotic systems. In this topology, the neighbourhood system at x consists of all supersets of the equivalence class $[x]_{\sim}$. (ii) For any subset A of the set X , the *A-inclusion topology on X* comprises \emptyset and every superset of A , while the *A-exclusion topology on X* are all subsets of $X - A$. Thus A is open in the inclusion topology and closed in the exclusion, and in general *every open set of one is closed in the other*. For a $x \in X$, the *x-inclusion neighbourhood \mathcal{N}_x* consists of all non-empty open sets of X which are the supersets of $\{x\}$, while for a point $y \neq x$, \mathcal{N}_y are the supersets of $\{x, y\}$. In the *x-exclusion topology*, \mathcal{N}_x are the non-empty open subsets of $\mathcal{P}(X - \{x\})$ that exclude x .

The possibility of generating different topologies on a set is of great practical significance in emergent, self-organizing systems because open sets define convergence properties of nets and continuity characteristics of functions that nature can play around with to its best possible advantage. The topologies introduced above play a key role in this programme.

Initial-and-Final Topology. The topological theory of convergence of nets and filters in terms of residual and cofinal subsets is fundamental in the development of this formalism, one of the goals being to understand the Second Law “dead” state of maximum entropy. We consider this problem as a manifestation of the change of the topologies induced by a non-bijective map $f : (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ to a state of *ininality* of *initial* and *final* topologies [19] of X and Y respectively. For a continuous f there may be open sets in X that are not inverse images of open sets of Y , just as it is possible for non-open subsets of Y to contribute to \mathcal{U} . When the triple $\{\mathcal{U}, f, \mathcal{V}\}$ is tuned in a manner such that neither is possible, the topologies so generated are the *initial* (smallest/coarsest) and *final* (largest/finest) topologies on X and Y for which $f : X \rightarrow Y$ is continuous.

For $e : X \rightarrow (Y, \mathcal{V})$, the *preimage or initial topology of X generated by e and \mathcal{V}* is³

$$\text{IT}\{e; \mathcal{V}\} \triangleq \{U \subseteq X : U = e^{-1}(V), V \in \mathcal{V}_{\text{comp}}\} \quad (1)$$

and for $q : (X, \mathcal{U}) \rightarrow Y$, the *image or final topology of Y generated by \mathcal{U} and q* is

$$\text{FT}\{\mathcal{U}; q\} \triangleq \{V \subseteq Y : q^{-1}(V) = U, U \in \mathcal{U}_{\text{sat}}\}. \quad (2)$$

A bijective *inal* function $f : (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ is a *homeomorphism*, and ininality for functions that are neither 1 : 1 nor onto generalizes homeomorphism; thus

$$U, V \in \text{IFT}\{\mathcal{U}; f; \mathcal{V}\} \Leftrightarrow \{f(U)\} = \mathcal{V} \text{ and } U = f^{-1}(V)$$

reduces to

$$U, V \in \text{HOM}\{\mathcal{U}; f; \mathcal{V}\} \Leftrightarrow \mathcal{U} = \{f^{-1}(V)\} \text{ and } \{f(U)\} = \mathcal{V}$$

for a bijective, open-continuous function. A homeomorphism $f : (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ renders the homeomorphic spaces (X, \mathcal{U}) and (Y, \mathcal{V}) topologically indistinguishable in as far as their geometrical properties are concerned. It is our hypothesis that the driving force behind the evolution of a system toward a state of dynamical homeostasis is the attainment of the ininal triple state (X, f, Y) for the system. The ininal interaction f between

³For a non-bijective function $f : (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$,

$$\begin{aligned} \mathcal{U}_{\text{sat}} &\triangleq \{U \in \mathcal{U} : U = f^{-1}f(U)\} \\ \mathcal{V}_{\text{comp}} &\triangleq \{V \in \mathcal{V} : V = ff^{-1}(V) = V \cap f(X)\} \end{aligned}$$

Here the “inverse” f^{-1} of f is defined by the projective conditions $ff^{-1}f = f$ and $f^{-1}ff^{-1} = f^{-1}$.

X and Y generates the smallest possible topology of f -saturated sets on X and the largest possible topology of images of these sets in Y and constitutes the state of uniformity represented by the maximum entropy of the second law of thermodynamics. Initality of f is simply an instance of non-bijective homeomorphism.

(B) Multifunctional Extension of Function Spaces: Graphical Convergence. The multifunctional extension is the smallest dense extension $\text{Multi}(X)$ of the function space $\text{map}(X)$. The main tool in obtaining the space $\text{Multi}(X)$ from $\text{map}(X)$ is a generalization of pointwise convergence of continuous functions to (discontinuous) functions [27] by a process of *graphical convergence of a net of functions* illustrated in Fig. 1. This defines neighbourhoods of $f \in \text{map}(X, Y)$ to consist of those functional relations in $\text{Multi}(X, Y)$ whose images at any point $x \in X$ lies not only arbitrarily close to $f(x)$ (as in the usual case of topology of *pointwise convergence*), but whose inverse images at $y = f(x) \in Y$ contain points arbitrarily close to x . Thus the graph of f is not only arbitrarily close to $f(x)$ at x in V , but must also be such that $f^{-1}(y)$ has at least branch in U about x such that f is constrained to cling to f as the number of points on the graph of f increases. Unlike for simple pointwise convergence, no gaps in the graph of the converged multi is permitted *not only on the domain of f , but on its range as well*.

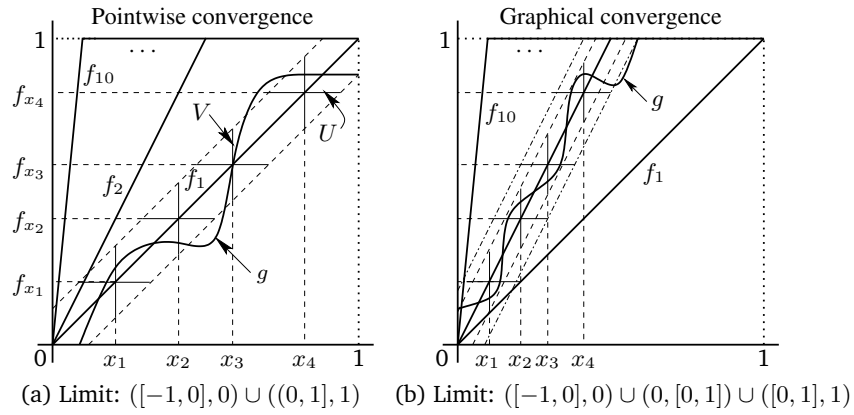


Figure 1: Pointwise and graphical biconvergence. Local neighbourhoods of $f_n(x) = \begin{cases} 0 & -1 \leq x \leq 0 \\ nx, & 0 < x \leq 1/n \text{ at } (x_i)_1^4 \\ 1, & 1/n < x \leq 1 \end{cases}$ with corresponding neighbourhoods (U_i) and (V_i) at $(x_i, f(x_i))$. The converged limit in (a) is a discontinuous function, in (b) it is a multifunction. It is this extension, from functional to general relations with its various ramifications, that constitutes the basis of chanoxity.

The usual topological treatment of pointwise convergence of functions is generalized to generate the boundary⁴ $\text{Multi}_{||}(X, Y)$ between $\text{map}(X, Y)$ and $\text{multi}(X, Y)$

$$\text{Multi}(X, Y) = \text{map}(X, Y) \cup \text{Multi}_{||}(X, Y) \cup \text{multi}(X, Y),$$

observe that the boundary of $\text{map}(X, Y)$ in the topology of pointwise biconvergence is a “line parallel to the Y -axis”.

Let $f: (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ be the iterative evolutionary unfolding of a noninjective map function in $\text{Multi}(X)$ and $P(f)$ the number of injective branches of f . Denote by

$$F = \{f \in \text{Multi}(X) : f \text{ is a noninjective function on } X \subseteq \text{Multi}(X)\}$$

the basic collection of noninjective functions in $\text{Multi}(X)$. For every α in some directed set \mathbb{D} , let F have the extension property

$$(\forall f_\alpha \in F)(\exists f_\beta \in F) : P(f_\alpha) \leq P(f_\beta).$$

Let a partial order \preceq on $\text{Multi}(X)$ be defined, for $f_\alpha, f_\beta \in \text{map}(X) \subseteq \text{Multi}(X)$ by

$$P(f_\alpha) \leq P(f_\beta) \Leftrightarrow f_\alpha \preceq f_\beta, (4)$$

with $P(f) := 1$ for the smallest f , define a partially ordered subset (F, \preceq) of $\text{Multi}(X)$. This is actually a preorder on $\text{Multi}(X)$ in which functions with the same number of injective branches are equivalent. The

⁴The boundary of A in X is the set of points $x \in X$ such that every neighbourhood N of x intersects both A and is complement $X - A$:

$$\text{Bdy}(A) \triangleq \{x \in X : (\forall N \in \mathcal{N}_x)((N \cap A \neq \emptyset) \wedge (N \cap (X - A) \neq \emptyset))\} \quad (3)$$

with \mathcal{N}_x the neighbourhood system at x .

existence of a maximal non-functional element in this process, obtained as the set theoretic “limit” of the net of functions with increasing nonlinearity, does not imply that it belongs to the functional chain as a fixed point. The net defines a corresponding net of increasingly multivalued functions ordered inversely by the relation

$$f_\alpha \preceq f_\beta \Leftrightarrow f_\beta^- \preceq f_\alpha^- \quad (5)$$

from which it follows that [27]

Chaotic map. Let A be a non-empty closed set of a compact Hausdorff space (X, \mathcal{U}) . A function $f \in \text{Multi}(X)$ is *maximally non-injective* or *chaotic* on A w.r.t. to \preceq if (a) for any f_i there exists an f_j satisfying $f_i \preceq f_j \forall i < j \in \mathbb{N}$, (b) the dense set $\mathcal{D}_+ := \{x : (f_\nu(x))_{\nu \in \text{Cof}(\mathbb{D})} \text{ converges in } (X, \mathcal{U})\}$ of isolated singletons is countable.⁵

The convergence implicit in the above definition is *graphical convergence* of the graphs of $(f_i)_i$ in $\text{Multi}(X)$ to allow for the extension of $\text{map}(X)$ to this more general class of relations. The existence and relevance of this extended class of one-to-many correspondences is fundamental to our consideration of chaos, complexity, and holism, and the bidirectionality of convergence with respect to the orthogonal axes of both (f_i) and its inverse (f_i^-) .

The collective macroscopic cooperation between $\text{map}(X)$ and its extension $\text{Multi}(X)$ generates the equivalence classes through fixed points and periodic cycles of f . As all points in a class are equivalent under f , a net or sequence converging to any must necessarily converge to every other in the set. This implies that the cooperation between $\text{map}(X)$ and $\text{Multi}(X)$ conspires to alter the topology of X to large equivalence classes. This dispersion throughout the domain of f of initial localizations suggests increase in entropy/disorder with increasing chaoticity; *complete chaos therefore implies the second law condition of maximum entropy enlarging the function space to multifunctions.*

(C) The Negative World \mathfrak{W} . Motivation: Competitive Collaboration. Of the axioms defining a vector space V , that of the additive inverse stipulating $\forall u \in V, \exists(-u) \in V, u + (-u) = 0$, comprises the crux of *competitive collaboration*. This *participatory* existence \mathbb{R}_- of \mathbb{R}_+ inducing a *reverse arrow* in \mathbb{R} , *competing collaboratively* with the forward arrow in \mathbb{R}_+ , serves to *complete* the structure of \mathbb{R} .

In a parallel vein, let W be a set such that for every $w \in W$ there exists a negative element $\mathfrak{w} \in \mathfrak{W}$ with the property that

$$\mathfrak{W} \triangleq \{\mathfrak{w} : \{w\} \oplus \{\mathfrak{w}\} = \emptyset\} \quad (8a)$$

defines the negative, or exclusion, set of W ⁶. Hence for all $A \subseteq W$ there is a neg(ative) set $\mathfrak{A} \subseteq \mathfrak{W}$ associated with (generated by) A that satisfies

$$\begin{aligned} A \oplus \mathfrak{G} &\triangleq A - G, \quad G \leftrightarrow \mathfrak{G} \\ A \oplus \mathfrak{A} &= \emptyset. \end{aligned} \quad (8b)$$

The pair (A, \mathfrak{A}) act as relative discipliners of each other in the evolving dissipation and tension, “undoing”, “controlling”, “stabilizing” the other. The exclusion topology of large equivalence classes in $\text{Multi}(X)$ successfully competes with the normal inclusion topology of $\text{map}(X)$ to generate a state of dynamic homeostasis in W that permits out-of-equilibrium complex composites of a system and its environment to coexist despite the privileged omnipresence of the Second Law. The evolutionary process ceases when the opposing influences in W and its moderator \mathfrak{W} balance in dynamic equilibrium by the generation of the ininal triple.

(D) Inverse And Direct Limits This abstract conceptual foundation of the existence of a complimentary negative world \mathfrak{W} for every real W permits participatory competitive collaboration between the two to generate self-organizing complex structures as summarized in Figs. 2a and 2b, see Refs. [11, 29] for the details. Briefly, the mathematical goal of chanoxity of establishing the existence of a disordering arrow for every dissipative ordering eventuality of large maximal equivalence classes of open sets through the attainment of the ininal topology, is additionally corroborated by the existence of these complimentary limits⁷, [11], possessing the following salient features.

⁵The residual and cofinal subsets

$$\text{Res}(\mathbb{D}) = \{\mathbb{R}_\alpha \in \mathcal{P}(\mathbb{D}) : \mathbb{R}_\alpha = \{\beta \in \mathbb{D} \text{ for all } \beta \succeq \alpha \in \mathbb{D}\}\}. \quad (6)$$

$$\text{Cof}(\mathbb{D}) = \{\mathbb{C}_\alpha \in \mathcal{P}(\mathbb{D}) : \mathbb{C}_\alpha = \{\beta \in \mathbb{D} \text{ for some } \beta \succeq \alpha \in \mathbb{D}\}\} \quad (7)$$

of a directed set \mathbb{D} are the basic ingredients of the topological theory of convergence of a net of functions.

⁶Notice that this definition is meaningless if restricted to W or \mathfrak{W} alone; it makes sense, in the manner defined here, only in relation to the pair (W, \mathfrak{W}) .

⁷For a family of sets $(X_\kappa)_{\kappa \in \mathbb{D}}$ the disjoint union is the set $\coprod_{\alpha \in \mathbb{D}} X_\alpha \triangleq \bigcup_{\alpha \in \mathbb{D}} \{(x, \alpha) : x \in X_\alpha\}$ of ordered pairs, with each X_α being canonically embedded in the union as the pairwise disjoint $\{(x, \alpha) : x \in X_\alpha\}$, even when $X_\alpha \cap X_\beta \neq \emptyset$. If $\{X_k\}_{k \in \mathbb{Z}_+}$ is an increasing family of subsets of X , and $\eta_{mn} : X_m \rightarrow X_n$ is the inclusion map for $m \leq n$, then the direct limit is $\bigcup X_k$.

For $\{X^k\}_{k \in \mathbb{Z}_+}$ a decreasing family of subsets of X with $\pi^{nm} : X^n \rightarrow X^m$ the inclusion map, the inverse limit is $\bigcap X^k$.

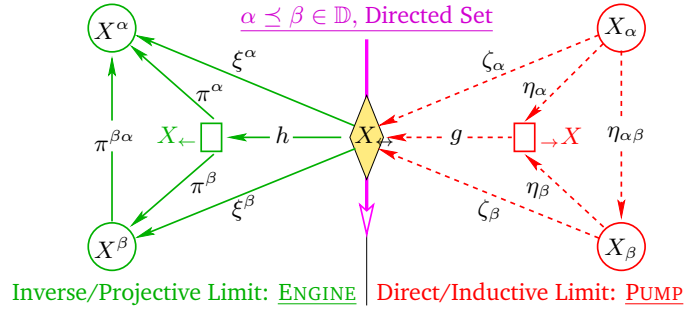


Figure 2a: Direct and inverse limits of direct and inverse systems $(X_i, \eta_{i\kappa})$, $(X^\iota, \pi^{\kappa\iota})$. Induced homeostasis is attained between the two adversaries by the respective arrows opposing each other as shown in the next figure where expansion to the atmosphere is indicated by decreasingly nested subsets.

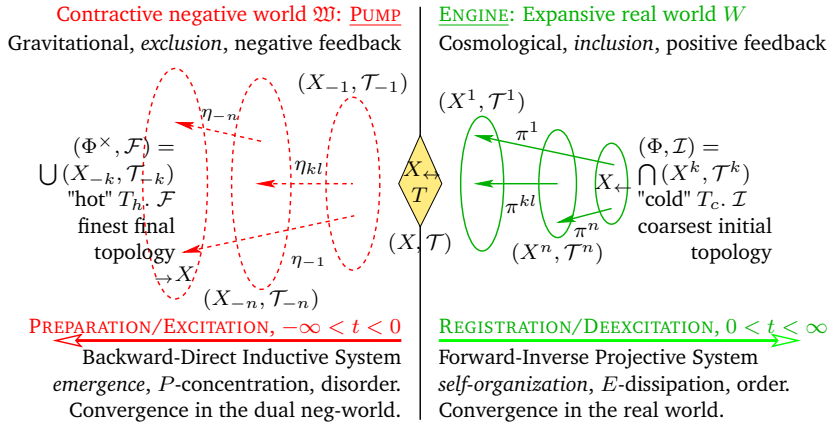


Figure 2b: Intrinsic arrows of time based on inverse-direct limits of inverse-direct systems. Intrinsic irreversibility follows since the dissipative forward-inverse arrow is the *natural* arrow in \mathbb{R}_+ equipped with the usual inclusion topology, while the backward-direct positive arrow of \mathbb{R}_- manifests itself as a dual “negative” exclusion topology in \mathbb{R}_+ . Notice that although E and P are born in $[T_h, T]$ and $[T, T_c]$ respectively, they operate in the domain of the other in the true spirit of competitive-collaboration. The entropy increases on contraction since the position uncertainty decreases faster than the increase of momentum uncertainty

For a given direction \mathbb{D} , the connecting maps π and η between the family of subsets $\{X^\alpha\}$ and $\{X_\alpha\}$ are oriented in opposition, the respective inverse and direct limits of the systems being X_{\leftarrow} and $\rightarrow X$ ⁸. The *mathematical existence* of these opposing limits, applicable to the problem under consideration, validates the arguments above and bestows on the gravitational disordering arrow with the sanction of analytic logic. Accordingly in Fig. 2b, reversal of the direction of \mathbb{D} to generate the forward and backward arrows completes the picture; observe the significant interchange of the relative positions of the two diagrams defining the homeostatic equilibrium $X_{\leftrightarrow}(T)$. If any of the two were to be absent, the remaining would operate within the full gradient $T_h - T_c$ and would be evolutionally harmless and impotent; in the homeostatic competitive case, however, the homeostatic condition T is generated and defined by the nonlinear competitive collaboration of these two opposites as will be seen below.

The inverse and direct limits are thus generated by opposing directional arrows whose existence follow from very general mathematical principles; thus for example existence of the union of a family of nested sets implies the existence of their intersection, and conversely. As a concrete example, Fig. 2b specializes to rigged Hilbert spaces $\Phi \subset \mathcal{H} \subset \Phi^\times$

$$\begin{array}{c} \xleftarrow{\text{Entropy-increasing Competitive}} \\ \Phi^\times \triangleq \bigcup_k H_{-k} \supset \cdots \supset H_{-i} \supset \cdots \supset H_{-1} \supset \mathcal{H} \supset \xrightarrow{\text{Entropy-decreasing, Collaborative}} \\ H^1 \supset \cdots \supset H^i \supset \cdots \supset \bigcap_k H^k \triangleq \Phi \end{array}$$

with Φ the space of physical states prepared in actual experiments, and Φ^\times are antilinear functionals on Φ that associates with each state a real number interpreted as the result of measurements on the state. Mathematically, the space of test functions Φ and the space of distributions Φ^\times represent definite and well-

⁸These limits are conventionally denoted \varprojlim and \varinjlim respectively

understood examples of the inverse and direct limits that enlarge the Hilbert space \mathcal{H} to the rigged Hilbert space $(\Phi, \mathcal{H}, \Phi^\times)$, with \mathcal{H} the homeostatic condition. Observe how without the mutual participation of either of these \leftrightarrow adversaries, emergence of the real number is impossible.

2.2 Physics of ChaNoXity: Stand-off between Individualism and Collectivism

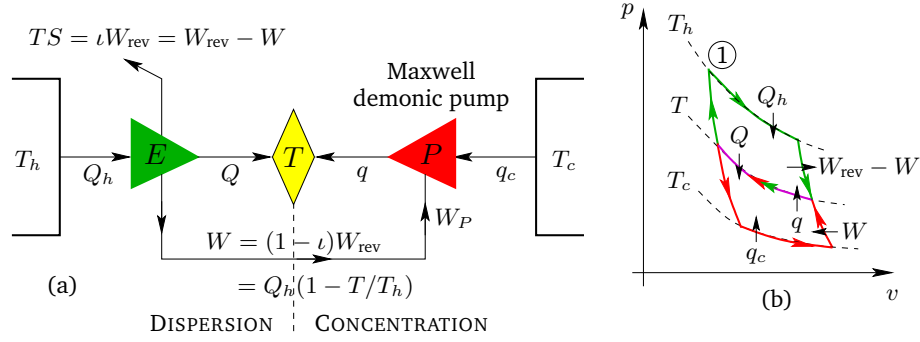


Figure 3: Reduction of the dynamics of opposites to an equivalent engine-pump thermodynamic system; $W_{\text{rev}} = Q_h(1 - T_c/T_h)$, $W(T) = Q_h(1 - T/T_h)$. The collaborative confrontation of Q and q , as reflected by their inverse relationship in Eq. (13) below with α the adaptability of these opposites in defining the complex α system, bestows on Q the possible interpretation of a “demand” that is met by the “supply” q in a bidirectional feedback loop that sustains, and is sustained by each other, in the context of the whole.

Assume that a complex adaptive system is distinguished by the complete utilization of a fraction $W(T) := [1 - \iota(T)]W_{\text{rev}} = (1 - T/T_h)Q_h$ of the work output of an imaginary reversible engine (T_h, E, T_c) with output $W_{\text{rev}} = (1 - T_c/T_h)Q_h$ to self-generate the pump P in competitive collaboration with E , Fig. 3. The *irreversibility factor*

$$\iota(T) \triangleq \frac{W_{\text{rev}} - W(T)}{W_{\text{rev}}} \in [0, 1] \quad (9a)$$

accounts for that part ιW_{rev} of available energy W_{rev} that cannot be gainfully utilized but must be degraded in increasing the entropy of the universe. Hence

$$\iota(T) = \left(\frac{T_R}{W_{\text{rev}}} \right) S(T) \quad (9b)$$

yields the effective entropy

$$S(T) = \frac{W_{\text{rev}} - W(T)}{T_R}. \quad (9c)$$

with reference to the induced temperature T . The self-induced pump decreases the temperature-gradient $T_h - T_c$ to $T_h - T$, $T_c \leq T < T_h$, generating dynamic stability to the system.

Let ι be obtained from

$$\begin{aligned} W_E &:= Q_h \left(1 - \frac{T}{T_h} \right) \triangleq W_P \\ &= Q_h(1 - \iota) \left(1 - \frac{T_c}{T_h} \right); \end{aligned} \quad (10)$$

hence

$$\iota(T) = \frac{T - T_c}{T_h - T_c} \quad (11a)$$

is formally similar to the quality

$$x(v) = \frac{v - v_f}{v_g - v_f} \quad (11b)$$

of a two-phase liquid-gas mixture with temperature T playing the role of specific volume v , where $T_h - T_c$ represents the internal energy that is divided into the non-entropic $T_h - T$ Helmholtz free (available) energy A internally utilized to generate the pump P and a reduced $T - T_c$ entropic dissipation by E , with respect to the induced equilibrium temperature T .

The generated pump is a realization of the energy available for useful, work arising from reduction of the original gradient $T_h - T_c$ to $T - T_c$. The irreversibility $\iota(T)$ is adapted by the engine-pump system such that the induced instability of P balances the imposed stabilizing effort of E to the best possible advantage of the system and the environment. Hence a measure of the energy that is unavailable to the system and must necessarily be dissipated to the environment is the *generalized entropy*

$$TS(T) = \iota(T)W_{\text{rev}} = W_{\text{rev}} - W(T) \quad (12a)$$

$$= U - A(T) \quad (12b)$$

which the system attains by adapting itself *internally* to a state of optimal competitive collaboration.

Figure 3 represents the essence of competitive collaboration: the dispersion of E is proportional to the domain $T - T_c$ of P , and the concentration of P depends on $T_h - T$ of E . Thus an increase in ι can occur only at the expense of P which opposes this tendency; reciprocally a decrease in ι is resisted by E . The induced pump P prevents the entire internal resource $T_h - T_c$ from dispersion at $\iota = 1$ by defining some $\iota < 1$ for a homeostatic temperature $T_c < T < T_h$, with E and P interacting with each other in the spirit of competitive collaboration at the induced interface T .

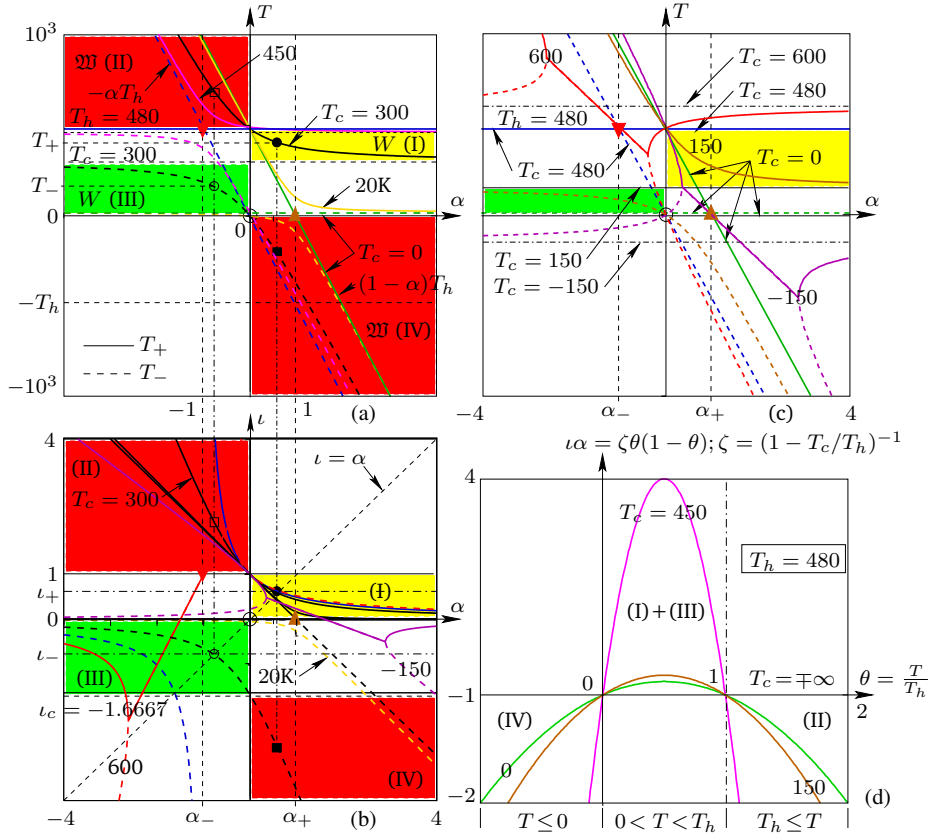


Figure 4: The interactive “participatory universe”, $T_h = 480\text{K}$; $\iota_c = -T_c/(T_h - T_c)$. The straight lines connecting the $T < T_c$ and $T > T_h$ segments in (b) and (c) correspond to complex roots. The colour-code is as in Fig. 3

Defining the equilibrium steady-state representing X_{\leftrightarrow} of homeostatic E - P adaptability $\alpha := \eta_E \zeta_P$, the equation of state of the participatory universe for $q(T) = (1 - \iota)Q(T_h - T_c)/(T - T_c) = [(1 - \iota)/\iota]Q$ with $Q(T) \triangleq Q_h - W(T) = Q_h - [1 - \iota(T)]W_{\text{rev}} = Q_h(T/T_h)$

$$\alpha(T) = \left(\frac{T_h - T}{T_h} \right) \left(\frac{T}{T - T_c} \right) \triangleq \frac{q(T)}{Q_h} = \frac{q(T)}{Q(T)} \left(\frac{T}{T_h} \right) \quad (13)$$

in the form $pv = f(T)$ where $p \equiv \zeta_P = 0$ at $T = 0$ and $v \equiv \eta_E$, be the product of the efficiency of a reversible engine and the coefficient of performance of a reversible pump. Fig. 4 for $T_h = 480\text{K}$ and $T_c = 300\text{K}$ shows that the engine-pump duality has the significant property of supporting two different states

$$T_{\pm}(\alpha) = \frac{1}{2} \left[(1 - \alpha)T_h \pm \sqrt{(1 - \alpha)^2 T_h^2 + 4\alpha T_c T_h} \right] \quad (14a)$$

$$= \begin{cases} ((1 - \alpha)T_h, 0) = (0, 0)_{\alpha=1}, & T_c = 0 \\ (T_h, -\alpha T_h) = (T_h, T_h)_{\alpha=-1}, & T_c = T_h \end{cases} \quad (14b)$$

for any value of α .

Fig. 4(b) suggests that the balancing condition

$$\iota(T) = \alpha(T) \quad (15)$$

defining the most appropriate equilibrium criterion

$$T_{\pm} = \frac{T_h(T_h + T_c) \pm (T_h - T_c)\sqrt{T_h^2 + 4T_cT_h}}{2(2T_h - T_c)} \quad (16a)$$

$$= \begin{cases} (0.5T_h, 0), & T_c = 0 \\ (T_h, T_h), & T_c = T_h \end{cases} \quad (16b)$$

directly determines the irreversibility of the interaction because the tendency to revert back to the original condition (small ι : predominance of pump P) implies large E - P adaptability α inviting E -opposition and the homeostasy of Eq. (15); see Fig. 10(a). Note that at $T_c = 0$, $T_- = T_c$ while at $T_c = T_h$, $T_+ = T_- = T_c$.

A complex system can hence be represented as

$$\underbrace{\begin{array}{c} \text{GRAVITATIONAL, BACKWARD-DIRECT} \\ \leftarrow P\text{-synthesis of concentration, disorder,} \\ \text{entropy increasing, bottom-up emergence } \rightarrow C \end{array}}_{\text{Individualism: Competitive "capital" } (\uparrow)} \oplus \underbrace{\begin{array}{c} \text{COSMOLOGICAL, FORWARD-INVERSE} \\ \rightarrow E\text{-analysis of dispersion, order, entropy} \\ \text{decreasing, top-down self-organization } C \leftarrow \end{array}}_{\text{Collectivism: Collaborative "culture" } (\downarrow)} \quad (17)$$

\Leftrightarrow Synthetic cohabitation of opposites C_{\leftrightarrow} ,

where \oplus denotes a non-reductionist sum of a top-down engine and its complimentary bottom-up pump that behaves in an organized collective manner with properties that cannot be identified with any of the individual components but arise from the structure as a whole: these systems cannot dismantle into their parts without destroying themselves.⁹ Analytic methods cannot simplify them as such techniques do not account for characteristics that belong to no single component but relate to the whole with all their interactions. Complexity is a dynamical, interactive and interdependent hierarchical homeostasis of P -emergent, disordering instability of competitive backward “capital”¹⁰ feedback in cohabitation with the adaptive, E -organized, ordering stability of collaborative forward feedback of “culture”¹¹ generating non-reductionist holism beyond the sum of its constituents.

<u>Forward-Inverse Projective arrow</u> (Natural direction in W)	<u>Backward-Direct Inductive arrow</u> (Natural direction in \mathfrak{W})
Top-down Engine E	Bottom-up Pump P
Dissipative: Self-organization	Concentrative: Emergence
Order: Entropy decreasing	Disorder: Entropy increasing
Collective: Collaborative	Individualistic: Competitive
<u>"Culture"</u>	<u>"Capital"</u>

Table 1: Holistic adverseries “capital” and “culture” of Nature.

Summarizing the evolutionary tension between “capital” and “culture” of Table 1, leads to the

⁹The definition of *cybernetics* as the study of systems and processes that “interact with themselves and produce themselves from themselves” by Louis Kauffman remarkably captures this spirit.

¹⁰“A factor of production which is not wanted for itself but for its ability to help in producing other goods; any form of wealth capable of being employed in the production of more wealth.” Wikipedia, the free encyclopedia.

Capital on its own therefore, is as impotent as sperm without egg.

¹¹“The set of shared attitudes, values, goals, and practices that characterizes an institution, organization or group; an integrated pattern of human knowledge, belief, and behavior that depends upon the capacity for symbolic thought and social learning.” Culture, according to Edward Tylor, “is that complex whole of knowledge, belief, art, morals, law, custom, and any other capabilities and habits acquired by man as a member of society”. Wikipedia, the free encyclopedia.

Culture on its own therefore, is as insatiated as egg without sperm.

Definition. Complexity. An open thermodynamic system of many interdependent and interacting parts is complex if it lives in synthetic competitive cohabitation with its self-induced negative dual in a state of homeostatic, hierarchical, two-phase equilibrium of collective top-down, dissipative, self-organizing, entropy-decreasing engine and individualistic bottom-up, concentrative, emergent, entropy-increasing pump dual, coordinated and mediated by the environment (“universe”).¹²

2.2.1 Complexity: Two-Phase Mixture of Bottom-Up Individualistic Competition and Top-Down Collective Collaboration: Critical and Triple Points

Consider Fig. 4 for the dual-pair (W, \mathfrak{W}) in relation to the formalization represented by Eqs. (11a, b). Fig. 4(a, b) defines four disjoint regions (I), (II), (III), (IV) characterized by the product

$$\begin{aligned} \iota\alpha(T) &= \frac{T(T_h - T)}{T_h(T_h - T_c)} \\ &\geq 0 \end{aligned} \quad (18a)$$

with positive values defining W in (I) and (III) and $\iota\alpha < 0$ representing W in (II) and (IV), refer Fig. 4(d). The significant point is the full specification of these regions in terms of the product and the direct linkages of region (I) with (II) through T_+ and of (III) with (IV) by T_- . Considering T_c as a variable with T_h given, produces the bounds of Eqs. (14b) and (16b) with the rather remarkable property that in the operational range

$$0 < T_c < T_h, \quad (18b)$$

T_+ and T_- are composed of bifurcated components of $(T_+ = (1 - \alpha)T_h, T_- = 0)$ at $T_c := 0$ and $(T_+ = T_h, T_- = -\alpha T_h)$, at $T_c := T_h$; thus T_{\pm} in this region are holistic expressions of themselves at the limiting values of 0 and T_h ; see Fig. 4(a).

The non-trivial range

$$T_h \leq T_c \leq 0$$

— meaningful only for $T_h \rightarrow +\infty$ — is graphed in Fig. 4(c) and (d). Of fundamental importance is the fact that the roots of Eq. (14a) form *continuous curves* in these regions, bifurcating as individual holistic components at $\alpha_{\pm} = \pm 1$: notice how at these values the continuous curves change character in disengaging from each other to form separate linear entities before “collaborating” once again in generating the profiles T_{\pm} in the operating range. These adaptations of the engine-pump are substantive in the sense that the specific α -values denote physical changes in the global behaviour of the system; they mark the *critical* and *triple points* of Fig. 5. The two-phase complex surface denoted by $\alpha = \iota$ is to be distinguished from the general Pv region $\alpha = \eta\zeta$. Since the ideal participatory universe satisfies a more involved *nonlinear* equation (13) compared to the simple *linear* relationship of an ideal gas, diagram 5(b) is more involved than the corresponding (a), with the transition at the triple point $\alpha_+ = 1$ displaying very definite distinctive features. While (b) clearly establishes that the triple point cannot be accessed from the $\iota = \alpha$ surface and requires a detour through the general $\alpha = \eta\zeta$, it also offers a fresh insight on the origin of the insular nature of the absolute zero $T = 0$.

Equation (13) and Fig. 5(b) show that the 2-phase individualistic-collective “liquid-vapour” region $\iota = \alpha$ is distinguished by the imposed constancy of α — and hence of the product Pv — just as P and T separately remain constant in Fig. 5(a). At the critical point $v_f = v_g$ for passage to second order phase transition, $T_c = T_h$ requires $T_+ = T_-$ which according to Eqs. (14b) and (16b) can happen only at $\alpha_- = -1$ corresponding to the (P_{cr}, T_{cr}) of figure (a). At the other unique adaptability of $\alpha_+ = 1$ at $T_c \rightarrow 0$, the system passes into region (IV) from (III) just as (I) passes into (II) as $T_c \rightarrow T_h$ at α_- . Observe from Eq. (14a) that

$$(T_c \rightarrow 0) \iff (T_h \rightarrow \infty) \quad (19a)$$

allows the self-organizing complex phase-mixture of collaboration and competition to maintain its state T as the condition of homeostatic equilibrium¹³ when $(T_+, T_-) = (0, 0)_{\alpha_+=1}$. Simultaneously however, because $T_c < T_h$,

$$(T_c \rightarrow T_h) \implies (T_h \rightarrow \infty) \quad (19b)$$

implies from Eq. (14b) that $(T_+, T_-) = (T_h, T_h)_{\alpha_-=-1}$ is also true. Hence

$$(\alpha_+)_{T_c=0} \sim (\alpha_-)_{T_c=T_h} \quad (19c)$$

¹²**Succinctly:** Homeostasis of the holistic adverseries of emergent “capital” and self-organizing “culture” defines a complex system.

¹³Does the melting of the arctic icebergs and the recent severe blizzards in Europe and USA indicate the veracity of Eq. 19a?

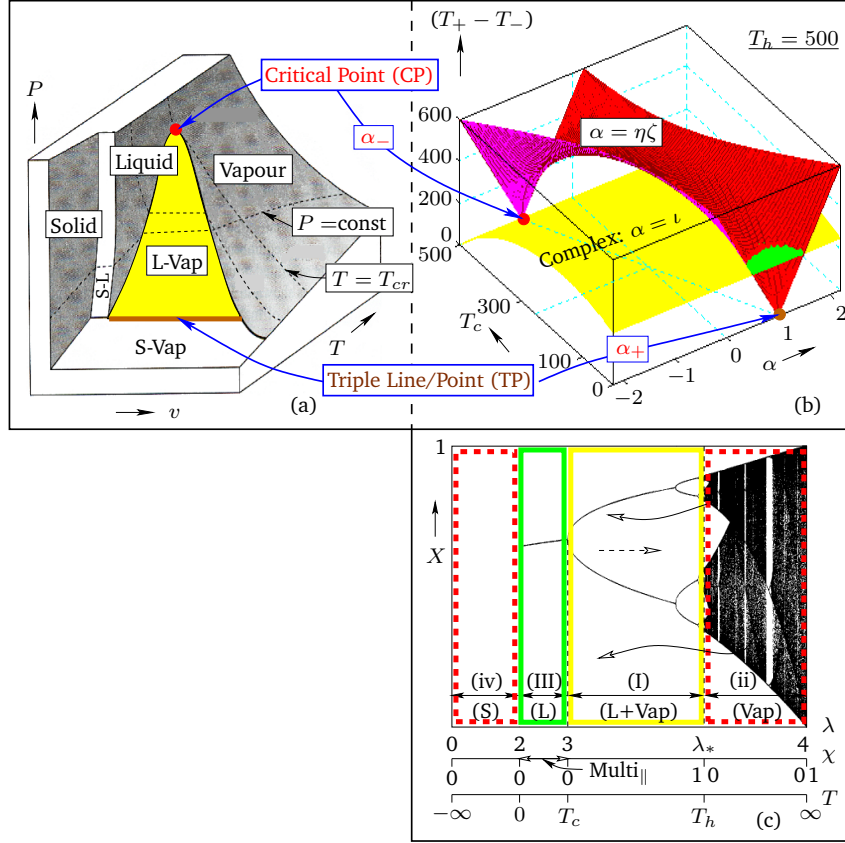


Figure 5: The 2-phase complex $\iota = \alpha$ region, (b) with critical point $T_c = T_h$ at $\alpha_- = -1$, yielding to α -dependent $\alpha = \eta\zeta$ at low T_c . The triple point $\alpha_+ = 1$, $T_c = 0$ is approachable only through this route. Compared to the normal transition of (a), self-organization in (b) occurs for $\alpha = Pv = \text{const}$, with P, v, T varying according to Eq. (13). $T_+ - T_- := \sqrt{T_h^2 + 4T_cT_h} \left(\frac{T_h - T_c}{2T_h - T_c} \right)$ at $\iota = \alpha$ is taken as an indicator of first-order-second-order transition because of Eqs. (14b) and (16b); (ii) and (iv) of (c) are the signatures of (II) and (IV) in W . The colour convention in (b) follows that of Fig. 3.

generates the equivalence

$$(T_+ - T_-)_{\alpha_+, T_c=0} = (T_+ - T_-)_{\alpha_-, T_c=T_h} \quad (19d)$$

providing an interpretation of the simultaneous validity of Eqs. (19a, b). The limiting consideration (19a) leaves us with two regions: (I) characterized by $\iota\alpha > 0$ of the complex real world W and (IV) of $\iota\alpha < 0$ of the negative world \mathfrak{W} . The three phases of matter of solid, liquid and gas of our perception manifests only in W , the negative world not admitting this distinction is a miscible concentrate in all proportions. The reciprocal implications (19a-d) at the big-bang degenerate singularity $\alpha_+ = +1$ at $t = 0$ [29], instantaneously causes the birth of the (W, \mathfrak{W}) duality at some *unique admissible* value of α for $0 < T_c < T_h$ and complexity criterion $\iota = \alpha$, breaking the equivalence $\alpha_+ \sim \alpha_-$ of Eq. (19c).

Figure 5(c) identifies the complex W on the bifurcation diagram of the logistic map $\lambda x(1-x)$ that we now turn to.

2.3 Philosophy of ChaNoXity: Both adversaries win, both lose

2.3.1 The Logistic Map $\lambda x(1-x)$: An Elementary Nonlinear Qubit

A correspondence between the dynamics of the engine-pump system and the logistic map $\lambda x(1-x)$, with the competitive, backward-direct iterates $f^i(x)$ corresponding to the “pump” \mathfrak{W} and the collaborative, forward-inverse iterates $f^{-i}(x)$ to the “engine” W , is summarized in Table 2. The two-phase complex region (I), $\lambda \in (3, \lambda_*)$, $T_+ \in (T_c, T_h)$, $\iota \in (0, 1)$, is the outward manifestation of the tension between the regions (I), (III) on the one hand and (II), (IV) on the other: observe from Eq. (14a) and Fig. 4 that at the environment $T_c = (0, T_h)$ the two worlds merge at $\alpha_{\pm} = \pm 1$ bifurcating as individual components for $0 < T_c < T_h$. The logistic map — and its possible generalizations — with its rising and falling branches denoted (\uparrow) and (\downarrow) , see Fig. 6, constitutes a perfect example of an elementary *nonlinear qubit*, not represented as a (complex)

linear combination: nonlinear combinations of the branches generate the evolving structures, as do the computational base $(1\ 0)^T$ and $(0\ 1)^T$ for the linear qubit. This qubit can be prepared efficiently by its defining nonlinear, non-invertible, functional representation, made to interact with the environment through discrete non-unitary time evolutionary iterations, with the final (homeostatic) equilibrium “measured” and recorded through its resulting complex structures.

The effective power law $f(x) = x^{1-\chi}$ [28] for

$$\chi = 1 - \frac{\ln \langle f(x) \rangle}{\ln \langle x \rangle}, \quad 0 \leq \chi \leq 1, \quad (20a)$$

$$\langle x \rangle \triangleq 2^N \xrightarrow{\lambda=\lambda_*} \infty \quad (20b)$$

$$\begin{aligned} \langle f(x) \rangle &\triangleq 2f_1 + \sum_{j=1}^N \sum_{i=1}^{2^{j-1}} f_{i, i+2^{j-1}}, \quad N = 1, 2, \dots, \\ &= \{(2f_1 + f_{12}) + f_{13} + f_{24}\} + f_{15} + f_{26} + f_{37} + f_{48} \end{aligned} \quad (20c)$$

and the hierarchical levels $(N = 1)$, $[N = 2]$, $\{N = 3\}$, with $\langle x \rangle$ the 2^N microstates of the basic unstable fixed points resulting from the $N + 1$ macrostates $\{f^i\}_{i=0}^N$ constituting the net feedback $\langle f(x) \rangle$, bestows the complex system with its composite holism. Hence

$$\chi_N = 1 - \frac{1}{N \ln 2} \ln \left[2f_1 + \sum_{j=1}^N \sum_{i=1}^{2^{j-1}} f_{i, i+2^{j-1}} \right] \quad (21)$$

is the measure of chanoxity, where $f_i = f^i(0.5)$, $f_{i,j} = |f^i(0.5) - f^j(0.5)|$, $i < j$, and

$$\chi = \iota = \alpha, \quad \lambda \in (3, \lambda_* := 3.5699456) \quad (22)$$

in Regions (I) and (III) can be taken as the definite assignment of thermodynamical purview to the dynamics of the logistic map for $\iota\alpha = \chi^2$, χ being the *measure of chanoxity*, Eq. (20a).

$\iota; T; \alpha$	$\lambda; \chi$	x_{fp}
$(-\infty, \iota_c]; (-\infty, 0]; [\infty, 0)$	$(0, 1], (1, 2]; 0$	$(\bullet, -), (\circ, -)$
$\iota\alpha < 0$: MULTIFUNCTIONAL SIMPLE \mathfrak{W} (IV: Individualistic)		
$(\iota_c, 0); (0, T_c]; (0, -\infty)$	$(2, 3); 0$	(\circ, \bullet)
$\iota\alpha > 0$: FUNCTIONAL SIMPLE W (III: Collective)		
$(0, 1); (T_c, T_h); (\infty, 0)$	$[3, \lambda_*); [0, 1)$	$(\circ, \bullet/\circ)$
$\iota\alpha > 0$: FUNCTIONAL COMPLEX W (I: Individualistic \oplus Collective)		
$[1, \infty); [T_h, \infty); [0, -\infty)$	$[\lambda_*, 4), 4; \{0, 1\}$	(\circ, \circ)
$\iota\alpha < 0$: MULTIFUNCTIONAL CHAOTIC \mathfrak{W} (II: Individualistic)		

Table 2: Emergence of the “Participatory Universe”, for $0 < T_c < T_h$ in W ; $\iota_c = -T_c/(T_h - T_c)$; putting dynamics and thermodynamics together. Region I of complex homeostacy in competition and collaboration is the holistic cohabitation of the opposites individualism and collectivism.

Table 2 demonstrates that the dynamics of the logistic map undergoes a discontinuous transition from the monotonically increasing $0 \leq \chi < 1$ in $3 \leq \lambda < \lambda_*$ of region (I) to a disjoint world at $\chi = 0$ in the fully chaotic $\lambda_* \leq \lambda < 4$ of (II), thereby *reducing chaos to effective linear simplicity*. Eq. (22), Fig. 4(a), (b) demonstrate that the boundary Multi_{\parallel} between $W := \text{map}$ and $\mathfrak{W} := \text{Multi}$ of the chaotic region II, $\lambda \in [\lambda_*, 4)$, occurs for $\chi = 0$, $\iota = \alpha \neq 0$ in equivalence with (III), for $T_c > 0$. As $T_c \rightarrow 0$, the boundary degenerates a point at $\alpha = 1$.

According to Table 2, $\chi = 0$ of (I) and $\chi = 1$ of (II) establishes the one-one correspondences ($\lambda \in (2, 3)$, $\lambda \in [\lambda_*, 4)$) at the boundary Multi_{\parallel} , and ($\lambda = \lambda_*$, $\lambda \geq 4$) at $T_c = T_h \rightarrow \infty$, Eq. (19a), of a boundaryless transition between these complimentary dual worlds. Hence $T_c \geq T_h$ is to be interpreted to imply $-\infty < T_c \leq 0$ of negative temperatures that define \mathfrak{W} .

Observe that quantum mechanics comprises the linear boundary between W and \mathfrak{W} .

Index of Complexity

Equation (15) for $\iota = \alpha$ leads to

$$\begin{aligned}\iota_{\pm} &= \frac{T_h - 2T_c \pm \sqrt{T_h^2 + 4T_h T_c}}{4T_h - 2T_c} \\ &= \begin{cases} (0.5, 0), & T_c = 0 \\ \pm \frac{1}{2}(\sqrt{5} \mp 1), & T_c = T_h \end{cases}\end{aligned}\quad (23)$$

at temperatures T_{\pm} of Eq. (16a) denoted as T_{\bullet} and T_{\circ} in Fig. 4(a). The complexity σ of a system is expected to depend on both the irreversibility ι and the interaction α ; thus the definition

$$\sigma_{\pm} \triangleq \frac{-1}{\ln 2} \begin{cases} \tilde{\iota}_{-}[\iota_{+} \ln \iota_{+} + (1 - \iota_{+}) \ln(1 - \iota_{+})] \\ \iota_{+}[\tilde{\iota}_{-} \ln \tilde{\iota}_{-} + (1 - \tilde{\iota}_{-}) \ln(1 - \tilde{\iota}_{-})] \end{cases}\quad (24)$$

with $\tilde{\iota}_{-} = \iota_{-}/\iota_{+} \in [0, 1]$, ensures the expected two-state, logistic-like, non-linear qubit ($\uparrow\downarrow$) signature at T_{+} and T_{-} .

Unification of thermodynamic and logistic qubit ($\uparrow\downarrow$) dynamics of the self-induced engine-pump system is achieved by extending (15) to (22). This identification of thermodynamic and dynamic properties in the evolution of a complex system by associating its dynamical degree $\chi := 1 - \frac{\ln \langle f(x) \rangle}{\ln \langle x \rangle}$ linearly increasing with λ with the thermodynamic competitive-collaboration α , focuses on the distinction between χ and complexity σ_{+} representing a homeostatic balance between dispersion and concentration.

2.3.2 Quantum Mechanics: A Linear Representation of Chaos

- *Bell's inequalities represent, first of all, an experimental test of the consistency of quantum mechanics. Many experiments have been performed in order to check Bell's inequalities; the most famous involved EPR pairs of photons and was performed by Aspect and co-workers in 1982. This experiment displayed an unambiguous violation of CHSH inequality and an excellent agreement with quantum mechanics. More recently, other experiments have come closer to the requirements of the ideal EPR scheme and again impressive agreement with the predictions of quantum mechanics has always been found. If, for the sake of argument, we assume that the present results will not be contradicted by future experiments with high-efficiency detectors, we must conclude that Nature does not support the EPR point of view. In summary, the world is not locally realistic.*
- *These profound results show us that entanglement is a fundamentally new resource, beyond the realm of classical physics, and that it is possible to experimentally manipulate entangled states. A major goal of quantum information science is to exploit this resource to perform computation and communication tasks beyond classical capabilities. Violation of Bell's inequalities is a typical feature of entangled states.*

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Is quantum mechanics indeed a general theory that applies to everything from subatomic particles to galaxies as it is generally believed to be, i.e., is Nature indeed governed by entanglements of linear superposition in Hilbert space or is it an expression of the nonlinear holism of emergence, self-organization, and complexity that we have constructed above? What is clear is that some basic structure of holistic “entanglement” is involved in the expressions of Nature; what is not so clear and is the subject of our present concern is whether this is linear quantum mechanical or nonlinear, self-organizing-emergent, and complex.

Composite systems in QM are described by tensor products of vector spaces, a natural way of putting linear spaces together to form larger spaces. If V, W are spaces of dimensions n, m , $A: V_1 \rightarrow V_2, B: W_1 \rightarrow W_2$ are linear operators, then $C := \sum_i \alpha_i A_i \otimes B_i$ on the nm -dimensional linear space $V \otimes W$ defined by $C(|v\rangle \otimes |w\rangle) = \sum_i \alpha_i (A_i |v\rangle \otimes B_i |w\rangle)$, together with the bi-linearity of tensor products, endows $V \otimes W$ with standard properties of Hilbert spaces inherited from its components, with the state space of the composite being the tensor product of the spaces of the components.

In quantum mechanics, the basic unit of classical information of the b (inary)(dig)it of either “on $|\uparrow\rangle$ ” or “off $|\downarrow\rangle$ ”, is replaced by the *qubit* of a normalized vector in two-dimensional complex Hilbert space spanned by the orthonormal vectors $|\uparrow\rangle := (1 \ 0)^T, |\downarrow\rangle := (0 \ 1)^T$. The qubit differs from a classical bit in that it can exist either as $|\uparrow\rangle$ or as $|\downarrow\rangle$ or as a superposition $\alpha |\uparrow\rangle + \beta |\downarrow\rangle$ ($|\alpha|^2 + |\beta|^2 = 1$) of both. The distinguishing feature in the quantum case is a consequence of the linear superposition principle that allows the quantum system to be in any of the 2^N basic states simultaneously, leading to the non-classical manifestations of interference, non-locality and entanglement.

Entanglement is the new quantum resource that distinguishes it so fundamentally from the classical in the sense that with the qubit, the degeneracy 2^N of composite entangled states is hugely larger than the $2N$ possibilities for classical systems. An immediate consequence of this is that for physically separated and entangled S and E in state $(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ for example, a measurement of $|\uparrow\rangle$ on S reduces/collapses the entangled state to the separable $|\uparrow\downarrow\rangle$ so that a subsequent measurement on E in the same basis always yields the predictable result $|\downarrow\rangle$; if $|\downarrow\rangle$ occurs in S then E will be guaranteed to return the corresponding reciprocal value $|\uparrow\rangle$. System $|E\rangle$ has accordingly been altered by local operations on $|S\rangle$, with a measurement on the second qubit always yielding a predictable complimentary result from measurements on the first. In the linear setting of quantum mechanics, multipartite systems modeled in 2^N -dimensional tensor products $\mathcal{H}_1 \otimes \dots \otimes \mathcal{H}_N$ of 2-dimensional spin states, correspond to the 2^N “dimensional space” of unstable fixed points in the evolution of the logistic nonlinear qubit. This formal equivalence illustrated in Fig. 6 while clearly demonstrating how holism emerges in 2^N -cycle complex systems for increasing complexity with increasing λ — the emergent 2^N -cycle are “entangled” in the basic (\uparrow) and (\downarrow) components as the system self-organizes to the graphically converged multifunctional limits indicated by the brown lines: the parts surrendering their individuality to the holism of the periodic cycles also focuses on the significant differences between complex holism and quantum non-locality.

The converged holistic behaviour of complex “entanglement” reflects the fact that the subsystems have combined nonlinearly to form an emergent, self-organized structure of the 2^1 , 2^2 and 2^3 cycles in Fig. 6(a), (b) and (c) that cannot be decoupled without destroying the entire assembly; contrast with the quantum entanglement and the notion of partial tracing for obtaining properties of individual components from the whole. Unlike the quantum case, the complex evolutions are not linearly superposed reductionist entities but appear as emergent, self-organized holistic wholes. In this sense complex holism represents a stronger form of “entanglement” than Bell’s nonlocality: *linear systems cannot be chaotic, hence complex, and therefore holistic*. While quantum non-locality is a paradoxical manifestation of linear tensor products, complex holism is a natural consequence of the nonlinearity of emergence and self-organization.

Nature uses the $2^{N \rightarrow \infty}$ multiplicities of chaos as an intermediate step in attaining states that would otherwise be inaccessible to it. Well-posedness of a system is an obviously inefficient way of expressing a multitude of possibilities as this requires a different input for every possible output. The countably many outputs arising from the non-injectivity of f for a given input is interpreted to define complexity because in a nonlinear system each of these possibilities constitute a experimental result in itself that may not be combined in any definite predetermined manner. This multiplicity of possibilities that have no predetermined combinatorial property is the basis of the diversity of Nature.

The reduced density matrix plays a key role in *decoherence*, a mechanism by which open quantum systems interact with their environment leading to spontaneous suppression of interference and appearance of classicality, involving transition from the quantum world of superpositions to the definiteness of the classical objectivity. Partial tracing over the environment of the total density operator produces an “environment selected” basis in which the reduced density is diagonal. This irreversible decay of the off-diagonal terms is the basis of decoherence that effectively bypasses “collapse” of the state on measurement to one of its eigenstates. This derivation of the classical world from the quantum is to be compared with nonlinearly-induced emergence of complex patterns through the multifunctional graphical convergence route of the type in Fig. 6. Multiplicities inherent in this mode illustrated by the filled circles, liberated from the strictures of linear superposition and reductionism, allow interpretation of objectivity and definiteness as in classical probabilistic systems through a judicious application of the Axiom of Choice: *To define a choice function is to conduct an experiment*. Because of the drive toward ininal topology of maximal equivalence classes of open sets at chaos, the selection by choice function refers to the analogue of continuous quantum probability of the Bloch sphere rather than the discrete or randomized classical probability. Non-local entanglement and interference, the distinguishing features of this distinction, are more pronounced and pervasive in nonlinear complexity than in linear *isolated and closed*, quantum systems, with its origins in the noninvertible, maximal ill-posedness of the dynamics of the former compared to the bijective, reversible unitary Schrodinger evolution of the later. This identifying differentiation of quantum non-locality and complex holism forms the basis of the following inferences.

Unlike in the quantum-classical transition, complex evolving systems are in a state of homeostasis with the environment with “measurement” providing a record of such interaction; probing holistic systems for its parts and components is expected to lead to paradoxes and contradictions. A complex system represents a state of dynamic stasis between the opposites of bottom-up pump induced synthesis of concentration, disorder, and emergence, and top-down engine dominated analysis of dissipation, order, and self-organization, the pump effectively deceiving the Second Law through entropy reduction and gradient dissipation. While quantum non-locality is a natural consequence of quantum entanglement that endows multi-partite systems with definite properties at the expense of the individual constituents, the effective power law $f(x) = x^{1-\chi}$ of Eqs. (20a),(b),(c) and the discussions of the effective linearity of the chaotic region in Table 2, suggests the

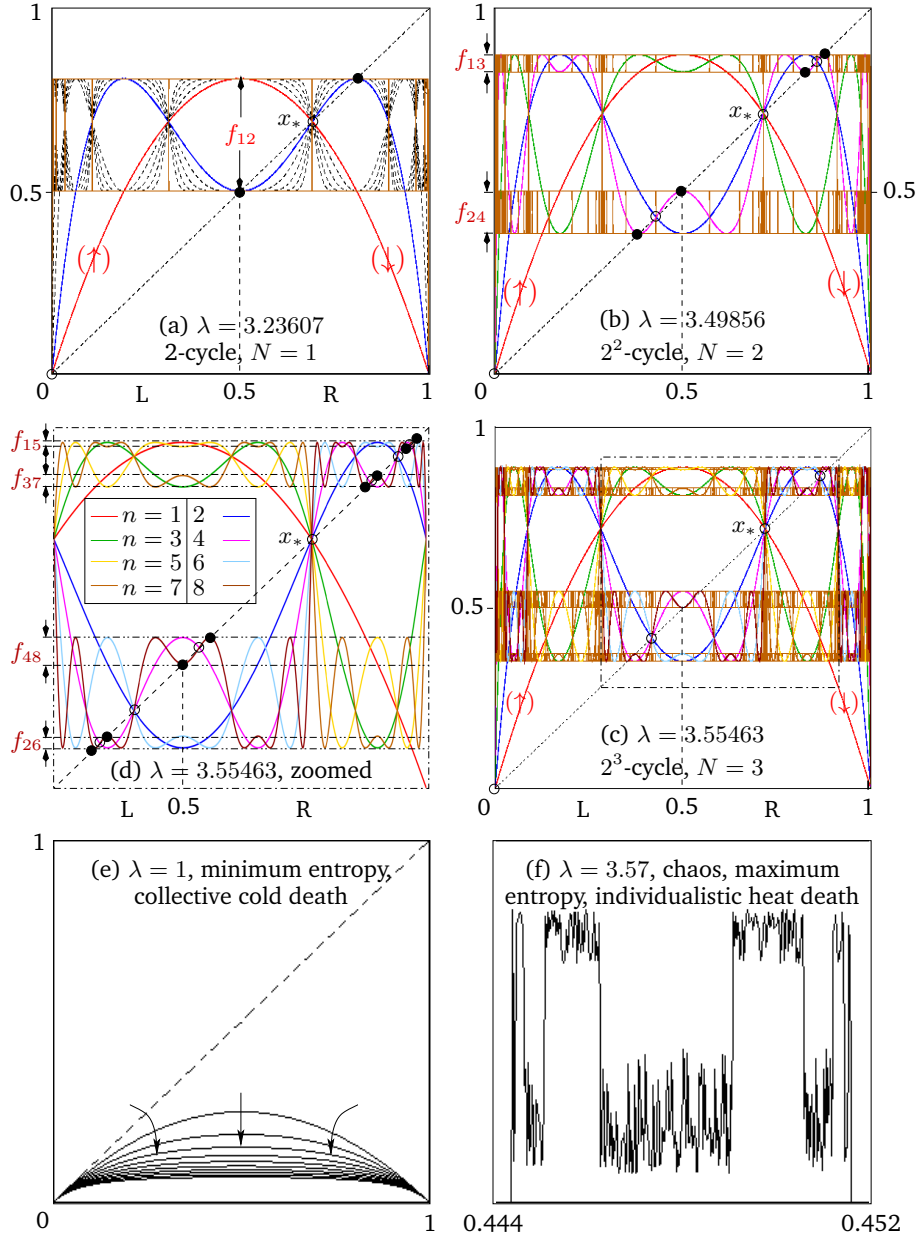


Figure 6: Complex entangled holism (a)-(d), generated by the logistic nonlinear qubit $f(x) = \lambda x(1-x)$. The effective nonlinearity $0 \leq \chi \leq 1$ of the representation $f(x) = x^{1-\chi}$ increases with λ , as the system becomes more holistic with an larger number of interacting parts of unstable fixed points shown unfilled, the stable filled points being the interacting, interdependent, components of the evolved pattern. The resulting holistic patterns of one, two, and three hierarchical levels are entangled manifestations of these observables, none of which can be independently manipulated outside of the collaborative whole. Direct iterates $f^i(x)$, (f), of increasing entropy, individualism, concentration comprises the backward gravitational arrow, the inverse iterates $f^{-i}(x)$, (e), of decreasing entropy, collectivism and dispersion is its opposite, with homeostasis of the graphically converged multifunctions comprising the dynamic equilibrium.

integration of quantum mechanics with chanoxity by identifying $\langle x \rangle = 2^N$ of Eq. (21) with the dimension of the resulting Hilbert space leading to the conjecture that *quantum mechanics is an effective linear representation* $\chi = 0$ of the fully chaotic, maximally illposed Multi_{||} boundary $\lambda_* \leq \lambda < 4$ that manifests itself — in equivalence with $\lambda \in (2, 3)$ — through a bi-directional, contextually objective, inducement of W in adapting to the Second Law of Thermodynamics. Quantum Mechanics resides at the interfacial boundary of W and \mathfrak{W} thereby possessing both the properties both of functional objectivity of the former and multifunctional ubiquity of \mathfrak{W} . The opposites of the (pump) preparation of the state and the subsequent (engine) measurement collaborate to define the contextual reality of the present. This combined with the axiom of choice allows the inference that *quantum mechanical “collapse” of the wave function is a linear objectification of the*

measurement choice function, the “measurement” process allowing the quantum boundary between the dual worlds of Table 2 to interact with the “apparatus” in W to generate the complex “reality” of the present ¹⁴.

Possibly the most ambitious projected utility of quantum entanglement and interference is of *quantum computers* [3]. Any two-level quantum system — like the ground and an excited states of an ion — that can be prepared, manipulated, and measured in a controlled way comprises a qubit, a collection of N qubits with its 2^N dimensional wave function in a Hilbert space representing a quantum computer. *Neglecting its coupling with the environment*, the *unitary* (hence invertible) time evolution of the computer is governed by the Schrodinger equation, with measurements disrupting this process. A *quantum computation* therefore consists of three basic steps: (i) preparation of the input state, (ii) implementation of the desired unitary transformation (*quantum gates*) acting on this state, and (iii) measurement of the output. In an ion-trap quantum computer for example, any linear array of ions constrained within a trap formed by static and oscillating electric fields is the quantum register. Ions are prepared in a specific qubit state by a laser pulses, the linear interaction between qubits being moderated by the collective vibrations of the trapped collection of ions.

The significant attributes of the programme for quantum computers in direct conflict with the defining features of chanoxism are the following. Isolation from the environment, invertible unitary interactions and the ability to selectively operate on constituent parts of the entangled state (of “Alice”, for example, who “shares an e-bit with Bob”) that in the ultimate analysis depend on the linear invertibility of unitary evolution, and superposition of quantum states. As none of these hold in complex holism, *being externally imposed classical interactions of the quantum system with its environment* and not self-generated, it can be hypothesized that *holistic computation, as the source of its linear quantum realization, is unlikely to be feasible*: unlike linear superpositions, any of the evolved holistic multifunctional entities in Fig. 6(a), (b), (c) cannot be decomposed or altered without adversely affecting the entire pattern with the inevitable consequence of critical instabilities impeding any serious, non-trivial, quantum computation.

The labeling of the interdependent, interacting, stable fixed points in Fig. 6 follows the following rule. The interval $[0, 1]$ is divided into two equal parts at $\frac{1}{2}$ with 0 corresponding to L and 1 to R. At any stage of the iterative hierarchy generated by the unstable (unfilled) points with the $f_{i < j}$ shown, the stable points are labeled left to right according to the prescription of Table 3 shown for $\langle f(x) \rangle = \{(2f_1 + f_{12}) + f_{13} + f_{24}\} + f_{15} + f_{26} + f_{37} + f_{48}\}$, the mean value of f according to Eq. (20c). This gives the symbolic representation

$$N = 1 \quad (0; 1) \quad (25a)$$

$$N = 2 \quad [(01, 00); (10, 11)] \quad (25b)$$

$$N = 3 \quad \{[(011, 010), (000, 001)]; [(101, 100), (110, 111)]\} \quad (25c)$$

for the self-organized, emergent levels corresponding to $N = 1, 2, 3$.

$f^i(0.5)$	L of unstable f.p.	R of unstable f.p.
convex up	0	1
convex down	1	0

Table 3: Rule for symbolic representation of the stable fixed points of Fig. 6 at each hierarchical level. Using this convention, these can be labeled left to right in (a), (b), (d) of the figure as $(0; 1)$, $[(01, 00); (10, 11)]$, and $\{[(011, 010), (000, 001)]; [(101, 100), (110, 111)]\}$ respectively.

As a specific example for $N = 2$, the complex “entangled” holistic pattern of Fig. 6(b) clearly demonstrates that the four components of Eq. (25b) cannot be decoupled into Bell states, being itself nonlinearly “entangled” rather than separated. The various operations historically performed on the respective qubits of the entangled pair to generate dense coding and teleportation ($N = 3$) for example, are not meaningful on the nonlinear holistic entities; in fact it is possibly not significant to ascribe any specific qubit to the individual members of the strings in Eq. (25b). These suggestive points of departure between linear quantum

¹⁴“While the linearity of quantum theory’s unitary process gives that theory a particular elegance, it is that very linearity (or unitarity) which leads directly to the measurement paradox. Is it so unreasonable to believe that this linearity might be an approximation to some more precise (but subtle) nonlinearity? ... Einstein’s theory explained these deviations, but the new theory was by no means obtained by tinkering with the old; it involved a completely radical change in perspective. This it seems to me is the kind of change in the structure of quantum mechanics that we must look towards, if we are to obtain the needed nonlinear theory to replace the present-day conventional quantum theory” [22].

nonlocality generated by external operations and nonlinear self-evolved complex holism calls for a deeper investigation that we hope to perform subsequently.

Nevertheless, the phenomenal success of linear quantum mechanics to “classify and predict the physical world” begs a proper perspective. Our hypothesis is that nature operates in accordance with chaoxity only in its “kitchen” that forever remains beyond our direct perception; what we do observe physically is only a linearized, presentable, table-top version of this complexity, through the quantum linear interface of $W - \mathfrak{W}$. This boundary between the dual worlds, of course, carries signatures of both, which seems to explain its legendary observational success.

2.3.3 Black Hole and Gravity: The Negative World and its Thermodynamic Legacy

(I) A Defining Example: The (W, \mathfrak{W}) , (Top-Down, Bottom-Up), (Particle, “Wave”) Duality Consider the two-state paramagnet of N elementary ($\uparrow\downarrow$) dipoles with magnetic field B in the $+z$ -direction. Then with μ the magnetic moment and

$$E = -N\mu B \tanh\left(\frac{\mu B}{kT}\right) \quad (26)$$

the total energy of the system, the corresponding expressions for temperature, entropy, and specific heat plotted in Fig. 7 displays the typical unimodal, two-state, ($\uparrow\downarrow$) character of S that admits the following interpretation. In the normalized ground state energy $E = -1$ of all spins along the B -axis, the number of microstates is 1 and the entropy 0. As energy is added to the system some of the spins flip in the opposite direction till at $E = 0$ the distribution of the \uparrow and \downarrow configurations exactly balance, and the entropy attains the maximum of $\ln 2$. On increasing E further, the spins tend to align against the applied field till at $E = 1$ the entropy is again zero with all spins opposing the field for a single microstate and *negative* T . The mainstream [2] relying on uni-directionality, holds that “all negative temperatures are hotter than positive temperatures. Moreover, the coldest temperature is just above 0K on the positive side, and the hottest temperatures are just below 0K on the negative side”. This view of \mathbb{R}_- of negative temperatures as a set of “super positives” is to be compared with what it really should be: the negative world \mathfrak{W} .

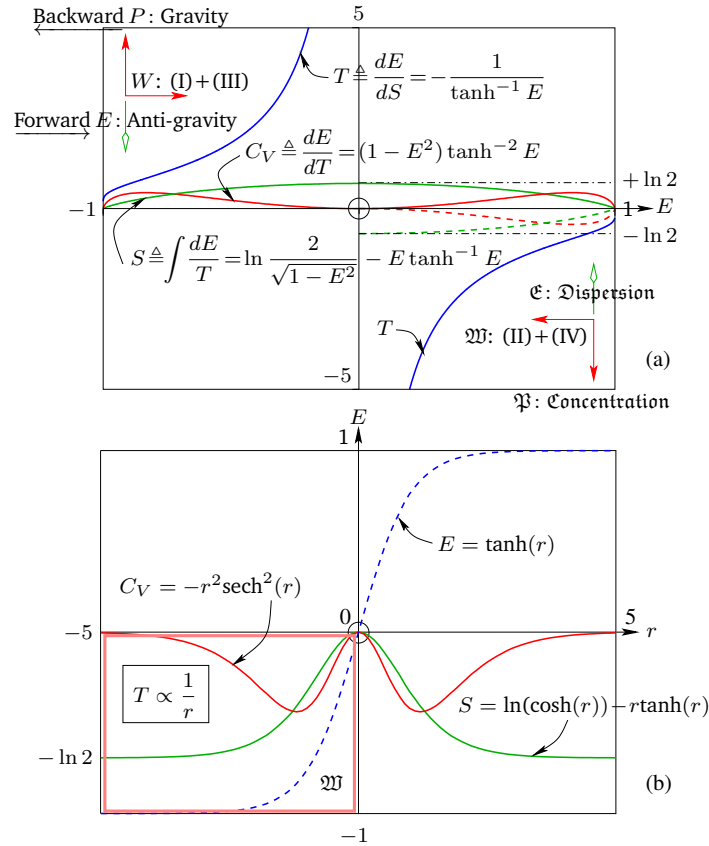


Figure 7: (a) Normalized ($N = \mu = B = 1 = k_B$) negative temperature, specific heat and entropy for a ($\uparrow\downarrow$) system. (b) The virial negative world \mathfrak{W} of negative specific heat and entropy.

Bidirectionally, quite a different interpretation based on the *virial theorem* relates the average kinetic energy of a system to its average potential energy, $2\mathcal{T} = -\sum_{k=1}^N \mathbf{F}_k \cdot \mathbf{r}_k$ where \mathbf{F}_k is the force on the k^{th} particle at \mathbf{r}_k . For power law potentials $\mathcal{V}(r) = cr^n$, the theorem takes the simple form

$$\mathcal{T} = \frac{n\mathcal{V}}{2}, \quad (\text{Virial Theorem}) \quad (27)$$

which for attractive $c = -1$ gravitational systems $n = -1$, reduces to

$$\mathcal{T} + E = 0, \quad E \text{ total energy.} \quad (28)$$

Since the potential energy decreases ($d\mathcal{V}/dr > 0$) faster than increase in kinetic energy ($d\mathcal{T}/dr < 0$) the total energy E decreases with bounding radius $dE/dr > 0$. With $\mathcal{T} \sim NT$, it follows

$$\mathcal{V} \sim -NT \Rightarrow T \propto \frac{1}{r}, \quad \frac{dT}{dr} < 0, \quad (29a)$$

and the gas gets hotter with shrinking radius. Hence

$$C_V \triangleq \frac{dE}{dT} = \left(\frac{dE}{dr} \right) \left(\frac{dr}{dT} \right) < 0 \quad (29b)$$

and, from $dS(r) \triangleq dE/T = -dT/T$, the entropy

$$S(r) \propto \ln r < 0, \quad r \rightarrow 0, \quad (29c)$$

becomes negative as $r \rightarrow 0$ gravitationally. It is clear from Eq. (29b) and Fig. 7 that only $E > 0$ with its direction reversed, $1 < E < 0$, can qualify for the gravitational region of negative specific heat and entropy. Hence

$$E(r) = \tanh(r), \quad -\infty < r < 0, \quad 1 < E < 0$$

applies only to the gravity-induced region of negative T and therefore of negative r , and

$$C_V(r) \triangleq -r^2 \text{sech}^2(r) \leq 0 \quad (30a)$$

$$= -r^2 + r^4 - \frac{2}{3}r^6 + \frac{17}{45}r^8 - \dots, \quad (30b)$$

$$S(r) \triangleq \int \frac{dE}{T} = \int C_V \left(\frac{dT}{dr} \right) \left(\frac{dr}{T} \right) = \int r \text{sech}^2(r) dr$$

$$= \ln \cosh(r) - r \tanh(r) \xrightarrow{r \rightarrow \infty} -\ln 2 \leq 0 \quad (30c)$$

$$= -\frac{1}{2}r^2 + \frac{1}{4}r^4 - \frac{1}{9}r^6 + \frac{17}{360}r^8 - \dots \quad (30d)$$

are both negative and even functions of r , as predicted by the arguments above. In the gravitationally collapsed region, therefore, entropy is proportional to r^2 (surface) *rather than to r^3 (volume) at small r — a characteristic feature of the black hole*. This most noteworthy manifestation of viriality in the dynamics of a ($\uparrow\downarrow$) system, of the natural appearance of negative r , can be taken as a confirmation of the existence of a negative, gravitationally collapsed world, that in fact constitutes a black hole. In this negative multifunctional dual \mathfrak{W} , where “anti-second law”¹⁵ requires heat to flow spontaneously from lower to higher temperatures with positive temperature gradient along increasing temperatures, the engine and pump exchange their roles *with disordering compression of the system by the environment — rather than entropy-decreasing expansion against it as in W — being the natural direction in \mathfrak{W}* . For an observer in W heat flows from higher *negative* temperatures to lower *negative* temperature.¹⁶

The opposing arrow of \mathfrak{W} translated to W , generate the full curves of Fig. 7; hence the entropy, specific heat and temperature are all positive as seen from \mathfrak{W} ($1 < E < 0$) dashed in the figure. In this framework, entropy increases with energy in \mathfrak{W} , rather than negative temperatures acting “as if they are higher than positive temperatures”: the temperature increases to infinity in \mathfrak{W} with $E \rightarrow 0_+$ as it does in W for $E \rightarrow 0_-$ with the *toroidal interconnection* between the complimentary dual worlds through the equivalences at $E = \pm 1$ and $T = \mp\infty$ allowing them to collaborate competitively as realized by the full curves. The maximum entropy of $\pm \ln 2$ occurs at $E = 0$ and the minimum at $E = \pm 1$ when all spins are aligned unidirectionally in single

¹⁵All qualifications are with respect to W .

¹⁶Refer Appendix (A) for some additional considerations.

microstates. *This manifestation of \mathfrak{W} in W produces the characteristic two-state ($\uparrow\downarrow$) signature of complexity and holism through the induced contractive manifestation of gravity*¹⁷

The pathological $T_h \leq T_c$ of (II) in the fully-chaotic region $\lambda \geq \lambda_*$, Fig. 4 and Table 2 where no complex patterns are possible, can now be understood with reference to Eqs. (19a-d) iff $T_h = \infty$ when (II) and (IV) merge in the single region of negative temperatures with its own “negative” dynamics in relation to W . Reciprocally at $T_c = 0$, region (III) vanishes and with $T_h = \infty$ leads to the two surviving $\alpha \geq 0$ portions of Table 2, one for $\iota\alpha < 0$ of the multifunctional (IV) of \mathfrak{W} and the other functional $\iota\alpha > 0$ of W (I). Since matter is born only in W as a gravitational materialization of the miscible mixture \mathfrak{W} , the boundary $\text{Multi}_{\parallel}(X)$ between the two worlds at $\chi = 0$, $\lambda \in [\lambda_*, 4) \sim \lambda \in (2, 3)$ is an expression of functional-particle, maximally-multifunctional-“wave” duality that is inaccessible from W because the equivalence at $E = \pm 1$ generates a passage between these antagonistic domains.

(II) Schwarzschild-de Sitter Metric: The Negative World \mathfrak{W} . The (self-induced) engine-pump system that forms the basis of our approach has a relativistic analogue in the Schwarzschild-de Sitter metric

$$ds^2 = -f_{\text{SdS}} dt^2 + f_{\text{SdS}}^{-1} dr^2 + r^2 d\Omega^2 \quad (31a)$$

where

$$f_{\text{SdS}} = 1 - \left(\frac{2GM}{c^2} \right) \frac{1}{r} - \left(\frac{\Lambda}{3} \right) r^2, \quad M > 0, \Lambda > 0 \quad (31b)$$

for an expansive cosmological opposition to gravitational compression. The zeros of f_{SdS} give the limiting values

$$r_{M,\Lambda} = \begin{cases} \frac{2GM}{c^2}, & \Lambda = 0 : \text{only gravitational contraction} \\ \sqrt{\frac{3}{\Lambda}}, & M = 0 : \text{only cosmological expansion} \end{cases} \quad (32)$$

of the Schwarzschild and cosmological radii r_M and r_Λ , respectively. Equation $f_{\text{SdS}} = 0$ solved for $M > 0$ and $\Lambda > 0$

$$M(\Lambda) = \frac{c^2}{2G} r \left(1 - \frac{\Lambda}{3} r^2 \right) > 0 \Rightarrow r < \sqrt{\frac{3}{\Lambda}} \triangleq r_\Lambda \quad (33a)$$

$$\Lambda(M) = \frac{3}{r^2} \left(1 - \frac{2GM}{c^2} \frac{1}{r} \right) > 0 \Rightarrow r > \frac{2GM}{c^2} \triangleq r_M; \quad (33b)$$

by imposing convenient bounds on the dynamics of the gravitational-cosmological tension, implies that

$$r_M < r < r_\Lambda, \quad M > 0, \Lambda > 0 \quad (33c)$$

corresponds to region (I) of the complex holistic world, Fig. 8. Reciprocally,

$$r_\Lambda < r < r_M, \quad M < 0, \Lambda < 0 \quad (33d)$$

is an indicator of the negative world \mathfrak{W} of negative temperature, Fig. 8 being a detailed representation of this equivalence: for $\Lambda = 1.38 \times 10^{-52} \text{ m}^{-2}$, the cosmological horizon $r_\Lambda \sim 1.47 \times 10^{26} \text{ m}$ is of the size of the observable universe, while with $M = 3.8 \times 10^{52} \text{ kg}$ as the mass of the observable universe, the gravitational Schwarzschild radius (event horizon) $r_M \sim 6.0 \times 10^{25} \text{ m} < r_\Lambda$.

The tilting of light cones for an expansionless $\Lambda = 0$ universe at the removable singularity of the Schwarzschild event horizon r_M that prevents future directed timelike or null worldlines reaching $r > r_M$ from the interior, corresponds to the passage to W through the $T_h = \infty$, $\alpha_- = -1$, Black Hole critical point, Fig 8(a), with $r_M > r \rightarrow 0$ collapsing gravitationally. At the other extreme of $M = 0$ as $r_\Lambda < r \rightarrow \infty$ expands without limit, $T \rightarrow 0$, denoting crossover to W through the $T_c = 0$, $\alpha_+ = 1$, Big Bang triple point. The gravitationally collapsed expression Eq. (29a)

$$T = \left(\frac{\hbar c}{k_B} \right) \frac{1}{r}, \quad (34a)$$

is the Hawking temperature¹⁸, while the entropy Eq. (30c), that for small r reduces to

$$S = - \left(\frac{c^3}{\hbar G} \right) r^2, \quad (34b)$$

¹⁷In GR, gravity is a manifestation of the curvature of spacetime geometry.

¹⁸With T negative in \mathfrak{W} , r must also be so.

is the negative of Hawking-Bekenstein entropy, but has the usual volumetric dependence of $\ln 2$ at full dispersion. The fully chaotic region $\lambda_* \leq \lambda \leq 4$ of the boundary Multi_{\parallel} between W and \mathfrak{W} , in equivalence with $2 < \lambda < 3$ of region (III) (see Fig. 8), is the “skin” of the gravitational black hole \mathfrak{W} at the physical singularity $r = 0$, identified as $M < 0$, $\lambda < 0$, in Eq. (33d). Gravity as we experience it in W , is the legacy of the thermodynamic arrow of \mathfrak{W} , see Fig. 7. The subsequent $T_c > 0$, $T_h < \infty$ exposition of Fig. 4 is responsible for the complex structures and patterns of Nature.

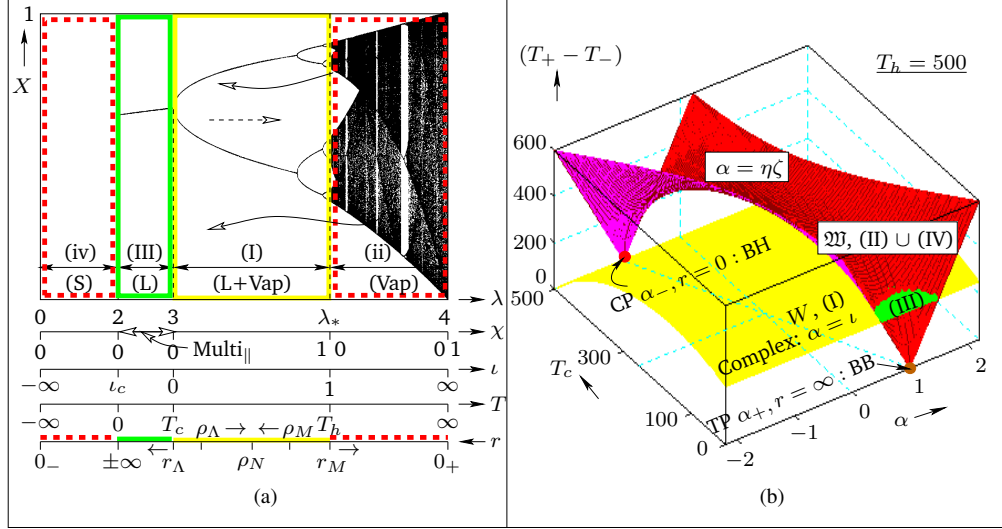


Figure 8: (a) Bifurcation profile of the Universe: Integration of the dynamical and thermodynamical perspectives. The complex phase (I) is a mixture of concentration and dispersion. $r_M = \frac{2GM}{c^2}$, $r_\Lambda = \sqrt{\frac{3}{\Lambda}}$ are the Schwarzschild and cosmological radii respectively with $\Lambda = 1.3793 \times 10^{-52} \text{ m}^{-2}$, $G = 6.6742 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$. $\ell_c = -T_c/(T_h - T_c)$.

The only real root of the cubic $f_{\text{sds}}(r) = 0$,

$$-r_{\text{sds}} = \frac{c^2}{(3GM\Lambda^2 c^4 + \sigma)^{1/3}} + \frac{(3GM\Lambda^2 c^4 + \sigma)^{1/3}}{c^2 \Lambda}, \quad (35a)$$

$$r_{\pm} = -\frac{r_{\text{sds}}}{2} \pm i\sqrt{3} \left[\frac{c^2}{2(3GM\Lambda^2 c^4 + \sigma)^{1/3}} - \frac{(3GM\Lambda^2 c^4 + \sigma)^{1/3}}{2c^2 \Lambda} \right] \quad (35b)$$

with $\sigma := \Lambda c^4 \sqrt{9G^2 M^2 \Lambda^2 - \Lambda c^4}$, has the significant property of being *negative*; the remaining complex pair merge to a single real value for

$$\sigma = 0 \Rightarrow 9G^2 M^2 \Lambda = c^4 \quad (36a)$$

at the Nariai radius

$$\begin{aligned} \rho_N = -\frac{r_{\text{sds}}}{2} &= \frac{1}{\sqrt{\Lambda}} \\ &= \frac{3GM}{c^2} \end{aligned} \quad (36b)$$

permitting f_{sds} to be factored as $f_{\text{sds}}(r) = -(\Lambda/3r)(r - \rho_N)^2(r + 2\rho_N)$. In general f_{sds} has two positive real zeros ρ_M and ρ_Λ satisfying

$$\begin{aligned} r_M < \rho_M < \rho_N = \frac{3GM}{c^2} < \rho_\Lambda < r_\Lambda, \\ \text{iff } 0 < \frac{3GM}{c^2} \sqrt{\Lambda} < 1 \end{aligned} \quad (36c)$$

with ρ_M monotonically *increasing* and ρ_Λ monotonically *decreasing* to the common value of ρ_N as $\Lambda \rightarrow c^4/9G^2 M^2$. Specifically requiring

$$\begin{aligned} \left(\frac{3G}{c^2} \right) M \sqrt{\Lambda} &> 1 \\ \Rightarrow M &\gtrsim 3.8 \times 10^{52} \text{ kg} \end{aligned}$$

as we do, prescribes M of magnitude of the mass of the observable universe.

While bypassing the significance of the negative root, the Nariai solution by equating the cosmological and mass horizons $\rho_\Lambda = \rho_M$ at the double root, deny the emergent feedback system of the two competitors of expansion (Λ) and compression (M) the privilege of collaborating with each other. Figure 8 and our approach by not insisting on this essentially unique reductionism, illustrate how gravity effectively moderates the Second Law dictate of *death* by allowing the system to “continually draw from its environment *negative entropy*”. The opposing effects at the degenerate singularities $\alpha_+ = 1$ and $\alpha_- = -1$ must be allowed to interact in order to generate the complex structure for $3GM\Lambda/c^2 > 1$ leading to a negative \mathfrak{W} with negative M . Recall from Fig. 8(a) and Table 2 that component (II) of \mathfrak{W} corresponds to the extreme nonlinearity $\lambda \geq \lambda_*$: this is the region where density of matter and curvature of spacetime are “so infinitely strong that even light cannot escape” This regularizes the singularity through collaborative competition of gravitational collapse and de Sitter expansion in W : mutual support of the two opposites generates the complex holistic structures of Fig. 8¹⁹. The negative real root of Eq. (35a) adds additional justification of the negative world through negative M ; observe however that Λ is not affected by this negativity of r , Eqs. (33a, 33b).

What is the significance of the negative root (35a) of negative mass M ? Our assertion in Sec. 2.2.1 that the three phases of matter are born only in W at $t = 0$ and have no meaning in \mathfrak{W} is supported by this distribution of the zeros, \mathfrak{W} being characterized fully by just the vacuum energy Λ . \mathfrak{W} induces in W two simultaneous effects (recall Figs. 2b and 7): its concentrative, individualistic “capital”-ist arrow of compression induces the expansive collaborative “culture”-d arrow of W with its own dispersion inducing the gravitational attraction in W . This is how Nature’s holism operates through unipolar gravity, with the concentration in \mathfrak{W} completing its bipolarity. Gravity is uniquely distinct from other known interactions as it straddles (W, \mathfrak{W}) in establishing itself, the other known forms reside within W itself.²⁰ It is this unique expression of the maximal multifunctional nonlinearity of \mathfrak{W} in the functional reality of W that is responsible for the inducement of “neg-entropy” effects necessary for the sustenance of life.

3 Conclusion: *Reality is not Flat*

In his remarkable explorations along *The Road to Reality*, Roger Penrose²¹ repeatedly stresses his conviction of “powerful positive reasons to believe that the laws of present-day quantum mechanics are in need of a fundamental (though presumably subtle) change”, basing his arguments on the “distinctly odd type of way for a Universe to behave” in the reversible unitarity of Schrodinger evolution \mathbf{U} being inconsistently paired with irreversible state reduction \mathbf{R} . This leads him to posit that “perhaps there is a more general mathematical equation, or evolution principle, which has both \mathbf{U} and \mathbf{R} as limiting approximations”, see footnote 14. In fact, “a gross time-asymmetry (is) a necessary feature of Nature’s quantum-gravity union”: gravity “just behaves differently from other fields”. Specifically, “there is some connection between \mathbf{R} and the Second Law”, with quantum state reduction being an *objectively real process* arising from the difference of gravitational self-energy E_G ²² between different space-time geometries of the quantum states in superposition. Thus, all observable manifestations in Nature are interpreted to be *always* gravity induced, quantum superpositions decaying into one or the other state.

This philosophical stance is operationally consistent with the foundations of our theory, recall Sec. 2.3.3 in particular, the details being however, conspicuously different. The homeostasy of top-down-engine and bottom-up-pump endows the state of dynamical equilibrium with the distinctive characteristic of competitively cohabitating opposites (Eq. 17) in its continual search for life and order. The reality of the natural world of *not* being in a “flat” [14] state of dispersive maximum entropy is infact the quest of open systems to stay alive by temporarily impeding this eventuality through self-organized competitive homeostasis. Hierarchical top-down-bottom-up complex holism does not support “flatness”; because of its antithetical stance toward self-organization and emergence: such a world is essentially a dead world. The survival of open

¹⁹“Matter tells spacetime how to bend and spacetime tells matter how to move” — J. A. Wheeler.

²⁰“Gravity seems to have a very special status, different from that of any other field. Rather than sharing in the thermalization that in the early universe applies to all other fields, gravity remained aloof, its degrees of freedom lying in wait, so that the second law could come into play as these begin to be taken up. Gravity just seems to have been different. However one looks at it, it is hard to avoid the conclusion that in those circumstances where quantum and gravitational effects must both come together, gravity just behaves differently from other fields. For whatever reason, Nature has imposed a gross temporal asymmetry on the behaviour of gravity in such circumstances.” [22]

²¹“The usual perspective with regard to the proposed marriage between these two theories is that one of them, namely general relativity, must submit itself to the will of the other. . . . Indeed the very name ‘quantum gravity’ that is normally assigned to the proposed union, carries the implicit connotation that it is a standard *quantum* (field) theory that is sought. Yet I would claim that there is observational evidence that Nature’s view of this union is very different from this! Her design for this union must be what, in our eyes, would be a distinctly non-standard one, and that an objective state reduction must be one of its important features.” [22]

²²Gravitational self-energy in a mass distribution is the amount of (binding) energy gained in assembling the mass from point masses dispersed at infinity.

living systems lies in its successfully guarding against this contingency through the expression of gravity as a realization of the multifunctional “quantum” \mathfrak{W} on the materially tangible W .

A socially significant remarkable example of this competitive collaboration is the open source/free software dialectics, developed essentially by an independent, dispersed community of individuals. Wikipedia as an exceptional phenomenon of this collaboration, along with Linux the computer operating system, are noteworthy manifestations of the power and reality of self-organizing emergent systems. How are these bottom-up community expressions of “peer-reviewed science” — with bugs, security holes, and deviations from standards having to pass through peer-review evaluation of the system (author) in dynamic equilibrium of competitive collaboration with the reviewing environment — able to “outperform a stupendously rich company that can afford to employ very smart people and give them all the resources they need? Here is a possible answer: Complexity. Open source is a way of building complex things” [20]. Note also that “the world’s biggest computer company (IBM) decided that its engineers could not best the work of an ad-hoc open-source collection of geeks (Apache Web server), so they threw out their own technology and decided to go with the geeks!” [14].

Which brings us to the main issue: Building anything, open-source or otherwise, requires investment of resources, financial and human. While the human incentive of open-sourcing for personal recognition through peer-review is a major deciding factor for the individual component, “collaborating for free in the open-source manner (as) the best way to assemble the best brains for the job” guarantees the collective ingredient needed for emergence of these complex systems that are far beyond the capacity of any single organization to handle. The blended model of revenue generation followed by most of the major open source groups contributes to the financial assets required for the self-generation of the backward pump as operationally viable, with the dispersive engine of a readily available market completing the engine-pump paradigm of chanoxity; *economics infact is about collectivism to inhibit human selfish individualism and promote evolution to a state of sustainable homeostatic, collective and societal holism*. The (social) unit “may be the individual or a collective of individuals. If it is a collective, could its behaviour be deduced from the sum of the behaviour of its components? Or could its behaviour be governed by other things than the sum of its components?” Unlike other customs in the analysis of social phenomena, the through and through individualistic character of neoclassical economics based almost entirely on the analysis of the behaviour of a single individual and his interaction with others “begins and ends with the individual, and sadly, there is barely any role to anything which is a reflection of the collective. . . . From the utility maximizing behaviour of individuals we derived the demand; from the profit maximizing behaviour of firms we derived the supply. The opposition of forces here is quite clear and well depicted by the demand and supply analysis (founded on Newtonian mechanics). Market is where the conflicting forces meet, and the most basic question is what might influence the outcome of an encounter between a consumer and a seller?” [31] Further insight into these economic considerations are considered in Appendix (B).

The science of collective holism is specifically addressed to issues such as these leading to an understanding of their true perspective.

APPENDIX

(A): Gravity and Entropy

Figure 9 adapted from Penrose [22], with the accompanying caption reproduced, is a vivid illustration of the special property of long range unipolar gravity²³, and further supports our arguments against considering negatives as “super positive”. Panels (a) and (b) are from [22] with the identification of (b) added. Recalling Figs. 2a, 2b, (a) and (c) represent the engine-pump duality expressed in Fig. 7(a).

²³“Whereas with a gas, the maximum entropy of thermal equilibrium has the gas uniformly spread throughout the region in question, with large gravitating bodies maximum entropy is achieved when all the mass is concentrated in one place — in the black hole.” [22]

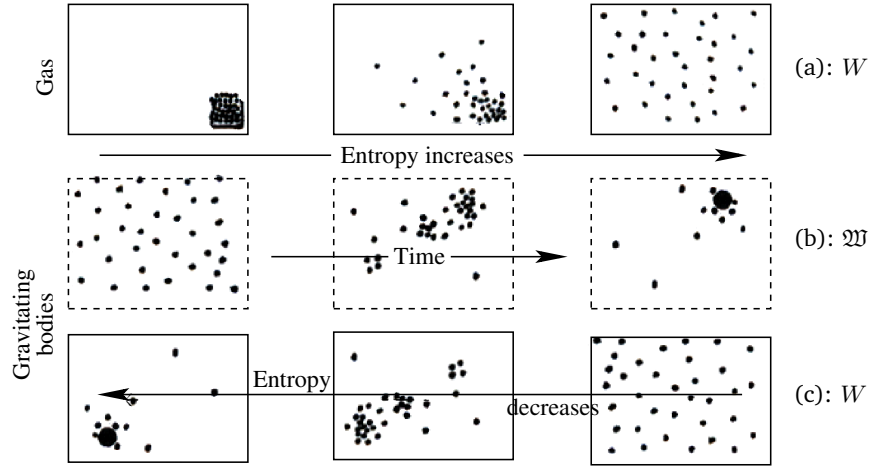


Figure 9: Increasing entropy with increasing time. (a) For gas in a box, initially tucked in one corner, entropy increases as the gas spreads throughout the box, finally reaching the uniform state of thermal equilibrium. (b) With gravity, things tend to be the other way about. An initial uniformly spread system of gravitating bodies represent a relatively low entropy, and clumping tends to occur as the entropy increases. From Penrose [22], p. 707.

(B): The Economic Turmoil: *Creative Destruction* of Economic Holism

Modern individualistic, neo-classical Western economics, is a static Newtonian equilibrium theory, where supply by the firm equals the demand of the consumer. Linear stability is central to this variant of economic thinking that has come under severe strain in recent times, *Economics Needs a Scientific Revolution*[5], *The Economy Needs Agent-Based Modelling* [13], *Meltdown Modelling* [6] reflecting some of the manifestations of this disillusionment. The linear mathematics of neoclassicalism is founded in calculus with maximization and constraint-based optimization being the ground rules, see [15] for example, that “Western economics became obsessed with” [23]. These Marshallian linear static models seeking to maximize utility for the consumer and profit for firms, as epitomized in Pareto optimality²⁴, Nash equilibrium²⁵, and Prisoner’s Dilemma²⁶ for example, work as might well be expected with reasonable justification, as long as its canonized axioms of

²⁴**Pareto Efficiency.** Given a set of alternative allocations for a collective of individuals, a change from one allocation to another that can make at least one individual better off without making any other worse, is called a Pareto improvement. An allocation is Pareto optimal when no further Pareto improvements can be made. The collectivism of Pareto efficiency can be expressed as

• A Pareto efficient situation is one in which any change to make someone better off is impossible without making somebody else worse off.

²⁵**Nash Equilibrium.** Let (S, f) be a game with n players, where S is the strategy set for player i , $S = S_1 \times S_2 \times \dots \times S_n$ is the set of strategy profiles and $f = (f_1(x), \dots, f_n(x))$ is the payoff function. Let x_{-i} be a strategy profile of all players except player i . When each player $i \in 1, \dots, n$ chooses strategy x_i resulting in strategy profile $x = (x_1, \dots, x_n)$ then player i obtains payoff $f_i(x)$: the payoff depends on the collective strategy of all. The collectivism of Nash equilibrium is

• A strategy profile $x^* \in S$ is a Nash equilibrium if no unilateral deviation in strategy by any single player is profitable for him:

$$\forall i, x_i \in S_i, x_i \neq x_i^* : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*). \quad (37)$$

²⁶**Prisoner’s Dilemma.** Two suspects A and B — each being interested only in maximizing his own advantage without any concern for the (collective) well-being of the other — are arrested by the police. The police have insufficient evidence for conviction, and, having separated the prisoners, visit each of them to offer the same deal. If one testifies for the prosecution against the other (“competes” (\downarrow) with the other) and the other remains silent (“collaborates” (\uparrow) with the other), the betrayer goes free (T) and the cooperating accomplice receives the full 10-year sentence (S). If both cooperate, they are sentenced to only six months (R) in jail for a minor charge. If each competes with the other, both receives a five-year sentence (P). Each prisoner must choose to compete with the other or to cooperate. How should the prisoners act?

• The Prisoner’s Dilemma can be summarized as follows, with (\uparrow), (\downarrow) denoting “collaboration”, “competition” of one with the other:

$A \downarrow B \rightarrow$	Collaborate (\uparrow)	Compete (\downarrow)
(\uparrow)	(6 mo (R), 6mo (R))	(10 ye (S), Free (T))
(\downarrow)	(Free (T), 10 ye (S))	(5 ye (P), 5 ye (P))

The Nash equilibrium of this game, which is not Pareto optimal ($\uparrow\uparrow$), is ($\downarrow\downarrow$) of 5 years each: competition dominates cooperation with competitors having a higher fitness than cooperators, compare Eq. 17 and Def. 2.2.

The pay-off matrix of benefits received by the parties defines a Prisoner’s Dilemma when $T > R > P > S$.

In the *Iterated Prisoner’s Dilemma*, when additionally $2R > S + T$, the participants have to choose their mutual strategy repeatedly with memory of their previous encounters, each getting an opportunity to “punish” the other for earlier non-collaboration. Cooperation may then arise as an equilibrium outcome, the incentive to defect being overcome by the threat of punishment leading to the possibility of a cooperative outcome. As the number of iterations increase, the Nash equilibrium tends to the Pareto optimum, the likelihood of cooperation increases, and a collective state of competitive-collaborating homeostasis emerges.

linearity of people with rational preferences acting independently with full and relevant information make sense. This framework of rationality of economic agents of individuals or company working to maximize own profits, of the “invisible hand” transforming this profit-seeking motive to collective societal benefaction, and of market efficiency of prices faithfully reflecting all known information about assets [5], can at best be relevant under severely restrictive conditions: “the supposed omniscience and perfect efficacy of a free market with hindsight looks more like propaganda against communism than plausible science. In reality, markets are not efficient, humans tend to be over-focused in the short-term and blind in the long-term, and errors get amplified, ultimately leading to collective irrationality, panic and crashes. Free markets are wild markets. Surprisingly, classical economics has no framework through which to understand ‘wild’ markets” (Bouchaud [5]). These “perfect world” models are meaningful under “linear” conditions only: “these successfully forecast a few quarters ahead as long as things stay more or less the same, but fail in the face of great change” (Farmer and Foley[5]), “as long as the influences on the economy are independent of each other, and the past remains a reliable guide to the future. But the recent financial collapse was a systemic meltdown, in which intertwined breakdowns . . . conspired to destabilize the system as a whole. We have had a massive failure of the dominant economic model” (Buchanan[5]).

These authors advocate an agent-based computer-modelling of economics (“the meltdown has shown that regulatory policies have to cope with far-from-equilibrium situations”), for simulating the interdependence and interactions of autonomous individuals with a view to assessing their effects on the system as a whole: the complex behaviour of adaptive system emerges from interactions among the components of the system and between the system and the environment. Individual agents are typically characterized as boundedly rational, presumed to be acting in what they perceive as their own interests such as economic benefit or social status, employing heuristics or simple decision-making rules. The computer keeps track of multiple agent interactions, monitoring a far wider range of nonlinear intercourse than conventional equilibrium models are capable of; “because the agent can learn from and respond to emerging market behaviour, they often shift their strategies, leading other agents to change their behaviour in turn. As a result prices don’t settle down into a stable equilibrium, as standard economic theory predicts” (Buchanan[5]).

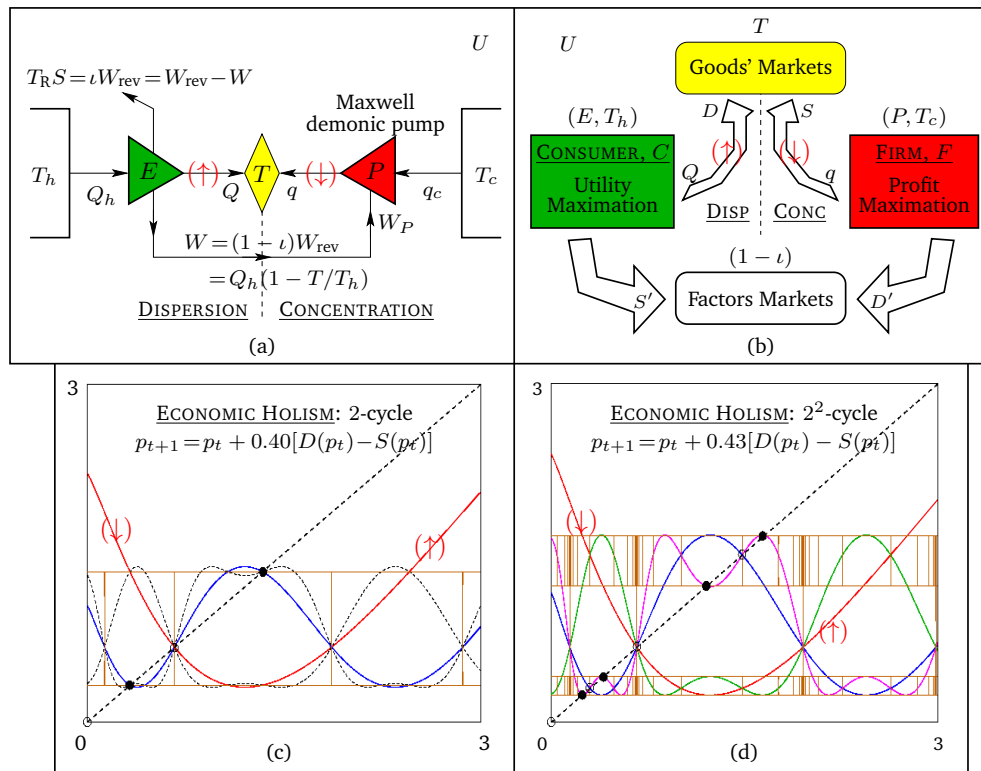


Figure 10: Economy as a complex system, U is the “universe”. (b) of neo-classical economics is adapted from Witztum [31]. According to this point of view, economics as the principal instrument of collective interaction in society, is to be distinguished from the exclusively individualistic stance of neo-classicalism. The Samuelson tatonnement of (c) and (d), to be compared with 6(a) and (b), show the emergence of economic complexity for nonlinear demand and supply profiles $D(p) = \frac{8.0}{1.1+p} - 1.75$, $S(p) = 10p^{1.5}e^{-p}$ respectively with p the commodity price.

This cellular automata²⁷ generated computer-graphics evolution of the economy strongly resembles the engine-pump realism of chanoxity as summarized in Fig. 10. The competitive collaboration of the engine and its self-generated pump is identified as the tension between the consumer with its dispersive collective spending engine (collaborative “culture”) in conflict with the individualistic resource generating pump (competitive “capital”) in mutual feedback cycles, attaining market homeostasis not through linear optimization and equilibrium of intersecting supply-demand profiles, but through nonlinear feedback loops that generate entangled holistic structures like those of Fig. 6. Supply and demand in human society are rarely independent: aggressive advertising for example can completely dominate the individual behaviour of these attributes. To take this into account, the interactive feedback between the opposites of engine consumption and pump production can be modelled as a *product* of the supply and demand factors that now, unlike in its static manifestation of neo-classicalism, will evolve in time to generate a condition of dynamic equilibrium, see Fig. 10 for the different evolution strategy of Samuelson tatonnement [8] for *nonlinear* Walrasian demand and supply profiles.

In the *linear* case, let $D(p) := 1 - \beta p$, $S(p) := \lambda p$, $\beta, \lambda > 0$, rescaled and normalized as $D(0) = 1$, $D(1) = 1 - \beta$, $S(0) = 0$ for $0 \leq p \leq 1$, be mappings on the unit square. Then supply and demand interact (mate) in the market via the shifted nonlinear qubit

$$f_{DS}(p) = \lambda p(1 - \beta p)$$

with a maximum $f_{DS}(p_m) = \frac{\lambda}{4\beta}$ at $p_m = \frac{1}{2\beta}$; note that at $\beta = 1$, f_{DS} reduces to the usual symmetric form $\lambda p(1 - p)$ and at $\beta = \frac{1}{2}$, $p_m = 1$. Since we are interested only in the range $\frac{1}{2} \leq f_{DS} \leq 1$ for possible complex effects, let the slopes of the two opposites be related by $\beta = 0.25\lambda$ for the expected $f_{DS}(1) = 0$ at $\lambda = 4$. The market clearing condition $D(p^*) = S(p^*)$ at $p^* = \frac{1}{\beta + \lambda} = \frac{4}{5\lambda}$ *apparently does not have any significance in the interactive evolution of $p_{t+1} = f_{DS}(p_t)$ with fixed point $p_{fp} = \frac{\lambda - 1}{\beta\lambda} = \frac{4(\lambda - 1)}{\lambda^2}$* , except at the uninteresting “solid-state” $\lambda = 1.25$ for $p^* = p_*$ (see Fig. 8(a)). The time evolution of the p_m -shifted, demand-supply qubit

$$f_{DS}(p) = \lambda p(1 - 0.25\lambda p), \quad \beta = 0.25\lambda \quad (38)$$

is similar to the symmetric $\lambda p(1 - p)$, except for a right-shift of p_m for $2 \leq \lambda < 4$.

The identification of demand D with mandatory heat rejection Q (by E) and of supply S with heat generated q (by P) requires some elucidation, recall footnotes 10 and 11. While the supply correspondence $S \Leftrightarrow q(T) := \alpha(T)Q_h$ in this positive-negative, auto-feedback loop responsible for a competitive market “capitalist” philosophy is fairly obvious, the demand analogy with $Q(T) := (T/T_h)Q_h$ is based on the argument that the confrontation of Q and q , bestows on Q a collective “cultural demand” that is met by individualistic “supply” of q in a bidirectional loop that sustains, and is sustained by each other, in the overall context of the whole. This collective and collaborative consumer demand complements, preserves, and nourishes the individualistic competitive supply q that constitutes the capitalist base of the firm²⁸.

Putting $Q(T) = q(T)$ for equality of demand and supply in Eq. (13), gives

$$\begin{aligned} T_{\pm} &= \frac{1}{2} \left(T_h - T_c \pm \sqrt{T_h^2 - 2T_h T_c + 5T_c^2} \right) \\ &= (403.21, -223.21), \end{aligned} \quad (39)$$

to be compared with the holistic $T_{\pm} = (406.09, 161.18)$ of Eq. (16a), with the limits

$$T_{\pm} = \begin{cases} (T_h, 0), & T_c = 0 \\ (T_h, -T_h), & T_c = T_h, \end{cases}$$

that are inconsistent with the holistic condition $\iota = \alpha$: the static equilibrium of supply and demand, as noted earlier and in contrast with the Samuelson tatonnement of Fig. 10, is possibly only a linear manifestation of economic complex holism.

The remarkable correspondence of this evolutionary profile with the logistic qubit interaction is far too pronounced to be dismissed as incidental. In situations as in the Prisoner’s Dilemma for example, the agents

²⁷Cellular automata (CA) are simple models of spatially extended decentralized systems comprising a number of individual component cells interacting with each other through local communications, with the state of a cell at any instant depending on the states of its neighbours. The division of CA into four classes [32] corresponding to the attractors of dynamical systems — Class 1: Stable Fixed Point, Class 2: Stable Limit Cycle, Class 3: Chaotic, Class 4: Complex — renders them attractive tools for graphical visualization of evolution like the emergence of altruistic or cooperative behaviour in Prisoner’s Dilemma [21] from classical Darwinian competition of second-law dispersion.

²⁸An example should clarify. The amount of sugar released in the blood by a supply of the “competitive capital” of energy input — if unchecked leading to heat death — is effectively countered in bidirectional homestatic demand of the “collaborative culture” of insulin — that remaining unchecked can only lead to a cold demise.

are in fact not free to take unilateral decisions but are in entangled holistic states of competitive collaboration with an accomplice — the two (unfilled) unstable fixed points of figure (d) — with the four possible outcomes of footnote 26 denoted by the (filled) stable fixed points, leading to the iterated dilemma corresponding to the converged holism of (d). When the entanglement is weak (linear) however, it is possible to consider the dilemma in terms of the Bell states in the base $(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)/\sqrt{2}$ resulting in the Nash equilibrium $(\downarrow\downarrow)$. Carrying this type of reasoning a step further, it is conceivable that globalization has effectively transformed the world economy into a single-celled monolith from its complex multi-cellular form, with the inevitable consequence that it is incapable of any further self-organization to a meaningful homeostatic form.

What is the economic analogy to the thermodynamics of open complex systems of Fig. 10? We suggest that economic profit

$$\pi(Y) = R - C(Y) \quad (40)$$

as the difference between total revenue R and total investment C , with Y the output of the economy, corresponds to irreversibility $\iota(T)$ of Eq. (11aa) that constitutes the foundation of chanoxity. With the specific mappings

$$\begin{aligned} R &:= W_{\text{rev}}(T_c) = \frac{T_h - T_c}{T_h} Q_h \\ C(Y) &:= W(T) = \frac{T_h - T}{T_h} Q_h \\ Y &:= T \end{aligned} \quad (41)$$

Q_h , the total infrastructural resources needed for the sustenance of a civil society that can support the consumer-firm interaction assumed to be suitably normalized, $\iota = \alpha$ holism requires from Eq. (16a) the very specific R – C relationship

$$C = \frac{3R - R\sqrt{5 - 4R}}{2(1 + R)}, \quad C(T) \triangleq 1 - \frac{T}{T_h}, \quad R(T_c) \triangleq 1 - \frac{T_c}{T_h} < \frac{5}{4} \quad (42a)$$

completely solves the economic holistic problem determining the “output” T , the profit being given by

$$\pi = \frac{2R^2 - R(1 - \sqrt{5 - 4R})}{2(1 + R)} \quad (42b)$$

Any (unutilized) profit unavailable for the benefit of the system can only increase the entropy of the universe by Eq. (12a). Uninhibited maximization of profit therefore corresponds to the Second Law dead-state of maximum entropy of turbulence, anarchy, and chaos ²⁹.

In orthodox neoclassical economics, there are two main kinds of recognized economic thinking — microeconomics that deals with small-scale economic activities such as that of the individual or company, and macroeconomics which is the study of the entire economy in terms of the total amount of goods and services produced, total income earned, the level of employment of productive resources, and the general behavior of prices. Meso-economics argues that there are important structures which are not reflected in the attributes of supply and demand curves, nor in the large economic measures of inflation, Gross Domestic Product, the unemployment rate, and other aggregate demand and savings measures. The argument is that the intermediate scale creates effects which need to be described using different measurements, mathematical formalisms and ideas. ChaNoXity represents a specific manifestation of this philosophical platform, the correspondences of Eq. (41) leading to the uncomfortable yet unavoidable diagnosis that the present social imbroglio — triggered by arguably ideologically motivated economic skulduggery — arises from the predictions of these models that “are’n’t even wrong, they are simply non-existent” [13]. However, since $\pi = 0$ corresponds to the reversible $\iota = 0$ quasi-static dead state, Nature’s future can only be ensured at the expense of its past: meaningful survival of the present depends on a careful and intentioned balance of the forward and backward arrows through the environmental resources of the system.

Nature is in fact a delicately balanced nonlinear complex of “capital” and “culture” representing the arrows of individualism and collectivism respectively.

²⁹ Adopting the point of view that the Second Law maximum entropy forward state of dissipation, degradation, and waste comprises “bad” while its opposite of enforced constructivism, usefulness and order defines “good”, the inescapable synthesis of our analysis is that Nature discards the high-entropy “bad” to make way for the low-entropy “good” in its dynamical quest of life. Paradoxically either, on its own, spells “death” and only their judiciously engineered intermingling can support and sustain Nature.

The Creative Destruction of Mesoeconomics

Economics, as the social science that examines how individuals use limited or scarce resources in satisfying their unlimited wants, is dominated by mainstream neoclassical economics that is plainly reductionist in nature and fiercely micro-individualistic, with societal macro-collectivism appearing merely as an aggregation of the former, motivated by unashamed aspiration of unlimited desires. Collectivism is mostly an assumed axiomatic imposition on the structure, without any inquiry on whether such predetermined equilibria do indeed ever exist. There is also the lurking suspicion that this version of modern economics “is sick. Economics has increasingly become an intellectual game played for its own sake and not for its practical consequences for understanding the economic world. Economists have converted the subject into a sort of social mathematics in which analytical rigour is everything and practical relevance is nothing” [4] that “bears testimony to a triumph of ideology over science” [30]. The over-arching camouflage of mathematical sophistry is the “essence of neoclassical economics, its response to criticism, and its remarkable capacity to turn explanatory failure into theoretical triumph” [1]. “What happened to the economics profession?” inquires Paul Krugman [17], “And where does it go from here?” He believes that “the economics profession went astray because economists, as a group, mistook beauty, clad in impressive-looking mathematics, for truth (with) a vision of capitalism as a perfect or nearly perfect system . . . in which rational individuals interact in perfect markets.”

That this “massive failure of the dominant economic model” cannot be understood, explained or remedied in its current incarnation is the considered opinion of many, even if this does not represent the mainstream. Kurt Dopfer et al. [10], following Schumpeter’s Legacy [9], analyse evolutionary economics in a new perspective of what they categorize Micro-Meso-Macro: “Our use of meso is more in the ontological, and therefore analytical, sense rather than in its classificatory sense. In our view, a meso is a thing that is made of complex other things (micro) and is an element of higher order things (macro). Meso is not in the intermediate sense of either classification or analysis of disequilibrium market structures, but rather in the specific sense of identifying and conceptualizing the dynamical building blocks of an economic system.”

Companies that once revolutionized and dominated new industries — for example, Xerox in copiers or Polaroid in instant photography — have had their profits fall and their dominance vanish as rivals launched improved designs or cut manufacturing costs. Any company that has achieved a strong position in the markets through its use of new inventory-management, marketing, and personnel-management techniques, can use its resulting lower prices to compete with older or smaller companies. Just as older behemoths were eventually undone by more innovative competitors, these trend-setters will face the same eventuality. The seemingly once dominant leaders may well find themselves antiquated through a process of “creative destruction” [26] that “incessantly revolutionizes the economic structure, incessantly destroying the old one, incessantly creating a new one”. This mutation from within is indeed the guiding doctrine of complex adaptive systems that constitutes the quintessence of our engine-pump philosophy. Thus the birth and death of stable-unstable fixed points in Figs. 6 and 10 encapsulating the “incessant mutation from within” of creative destruction, embraces the view of economics as the “science that studies the causes and consequences of the behaviour of many individuals dealing with commodities in a macroscopic system” [9]. Evolutionary economics as an inquiry into the question of how economic activities of many individuals are coordinated and change over time, examining the dialectics of competitively collaborating human interaction of “creative destruction”, is a significant step towards the understanding of “wild markets” and real phenomena. This of course is conspicuously absent from neoclassical analysis that “begins and ends with the individual, and sadly, there is barely any role to anything which is a reflection of the collective.”

According to the Schumpeterian vision of “the evolutionary response to the thermodynamic challenge is knowledge. . . . The hallmark of knowledge is that it can generate new knowledge which in turn generates new knowledge and so forth, self-perpetuating a continuous path of cumulated knowledge growth” needed to counter the inevitable entropic loss, the agent responsible for introducing change through novelty is an “entrepreneur”. The entrepreneur participates in the evolution not just passively (as his neoclassical counterpart) but more importantly in an active fashion, initiating a positive-negative feedback that results not just in a mere extension of a previous structure but through the novelty of emergence and self-organization at the “generic level”. By engaging actively in the economic process, “various building blocks are added one after another to an existing corpus, (which) also implies that the whole structure and fundamental characteristics of that corpus changes” [9], compare the stable periodic cycles of Figs. 6, 10.

In economic theory the process that generates the new structures from the old is called meso signifying an intermediate hybrid of micro and macro. The Schumpeterian entrepreneur who “carries out an innovation (micro) that are adopted and imitated by a population of followers (meso) thereby destroying the existing structure of the economy (macro) leads to an elementary unit composed of on the one hand, an idea or generic rule, and on the other many physical actualizations of it” [9] is a physical actualization of the increasing multifunctional ill-posedness associated with the time evolution of the logistic map. The bimodal

nature of the elementary unit breaks the traditional micro-macro dichotomy and by introducing meso leads to a new framework of micro-meso-macro [9].

In contrast with the implied linearity of Schumpeter's canvas, the strong nonlinearity inherent in chanoxity [29] adds a new dimension to the details of Schumpeter's vision as analysed and critiqued in Dopfer [9]. Extending this analogy, it is fair to claim that chanoxity constitutes a generic competitively-collaborating foundation for creative destruction, with the present collapse of the economic base having been engineered through a process of gradual decimation of the self-induced feedback mechanisms that support the existence of open thermodynamic systems.

Nonlinear self-organization and emergence are fascinating demonstrations of dynamical homeostasis of opposites, apparently the source and sustenance of Nature's diversity.

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